When Do Subjective Expectations Explain Asset Prices?

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January 2022

ABSTRACT

We present a method for determining whether errors in expectations explain asset pricing puzzles without imposing assumptions about the mechanism of the error. Using accounting identities and survey forecasts, we find that errors in expected long-term inflation and short-term nominal earnings growth explain price variation, return predictability, and the rejection of the expectations hypothesis for aggregate stock and bond markets. Errors in expected short-term inflation and long-term nominal earnings growth play no role. The relevant errors are consistent with mistakes about both the persistence of forecasted variables and the response to surprises. A fundamental extrapolation model successfully replicates these findings.

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*This paper was previously circulated under the title "Real Cash Flow Expectations and Asset Prices." Ricardo De la O is with the Marshall School of Business, University of Southern California. Sean Myers is with the Wharton School at the University of Pennsylvania. We thank Nicholas Barberis, Jules van Binsbergen, Gideon Bornstein, Laura Castillo-Martinez, David Hirshleifer, Rafael La Porta, Karen Lewis, Ernest Liu, Lars Lochstoer, Hanno Lustig, Alessandra Peter, Monika Piazzesi, Martin Schneider, Andrei Shleifer, Selale Tuzel, Michael Weisbach, Motohiro Yogo, and seminar participants at the NBER Behavioral Finance Spring 2021 Meeting, the USC Marshall School of Business and the Wharton School for valuable comments. Sean Myers gratefully acknowledges financial support from the Peter G. Peterson foundation. Email addresses: rdelao@marshall.usc.edu, semyers@wharton.upenn.edu.

Expectations of future fundamentals are a crucial part of asset pricing, as prices represent discounted expectations of future real cash flows. Under standard rational expectations (RE), investors' expectations match the objective probability distribution observed by an econometrician, which has led to many puzzles regarding price movements and future returns. However, RE imposes a huge number of restrictions on expectations and, unsurprisingly, many deviations from RE have been documented in subjective expectations.[1] While these systematic errors are by themselves an important challenge to the RE framework, they may or may not help in resolving asset pricing puzzles. This raises the question, when do deviations from RE help to explain asset prices?

In this paper, we document the importance of subjective expectations of *long-term inflation* and *short-term nominal earnings growth* for explaining aggregate bond and stock markets. Using accounting identities, we derive a diagnostic test on subjective expectations which (i) provides a necessary and sufficient condition for deviations from RE to explain price movements and return puzzles and (ii) is easily implementable. We apply this test to survey expectations to identify where the deviations from RE relevant for asset pricing do or do not appear. Though all of the expectations contain systematic errors, only expectations of long-term inflation and short-term nominal earnings growth satisfy this diagnostic test, while expectations of short-term inflation and long-term nominal earnings growth do not. The relevant errors are consistent with mistakes about both the persistence of the forecasted variables and the informativeness of recent surprises. Based on these results, we estimate a structural model of fundamental extrapolation where agents price assets using biased expectations of real cash flows which successfully replicates our findings.

We first establish a diagnostic test that determines whether deviations from RE in subjective expectations of real cash flows explain asset price movements, stock return predictability, and the rejection of the expectations hypothesis. The condition is that forecast errors for these expectations must be predictable using current prices. Importantly, not all deviations from RE satisfy this condition, meaning that it serves as a useful tool for disciplining which systematic errors are most relevant for explaining prices. Because this condition is derived from accounting identities, it is general and holds for any deviation from RE, e.g., learning, rational inattention, bounded rationality, and behavioral biases. For stocks, predictable forecast errors for real earnings growth cause price ratios to predict future returns even if discount rates are constant. For bonds, predictable forecast errors for inflation cause holding returns to be excessively volatile relative to the expectations hypothesis, even if term premia are constant.

We then document the importance of subjective expectations of long-term inflation and

[1] See Barberis and Thaler (2003) for a summary.

short-term nominal earnings growth using survey forecasts of CPI inflation and nominal earnings growth for the S&P 500. For short-term inflation and long-term nominal earnings growth, expectations do not comove any more or less with prices than future realizations comove with prices. As a result, prices do not predict forecast errors.[2] However, for long-term inflation and short-term nominal earnings growth, expectations comove substantially more with prices than future realizations do. This creates large forecast errors which satisfy the diagnostic test. Note that errors even for short-term nominal earnings growth can have a large impact on prices, as a change in one-period growth affects the level of all future nominal earnings. Since the diagnostic test is based on comovements, we are able to decompose the test results to show that the large comovement of long-term inflation expectations with prices is explained by the high believed persistence of inflation and that the large comovement of short-term nominal earnings growth expectations with prices is explained by the response to nominal earnings growth surprises.

Because of these systematic errors, subjective expectations of real cash flows closely match observed aggregate bond yields and stock prices.[3] Specifically, we calculate fundamental values for the ten-year zero-coupon Treasury yield and the S&P 500 price-earnings and price-dividend ratios based on the survey forecasts and constant discount rates. Over our 1976-2018 sample, these fundamental prices closely match the observed prices, with regression coefficients near 1 and high R^2's of 66%, 81%, and 79%, respectively.[4] A direct corollary to this finding is that discount rates play only a secondary role in price movements. Even if we attribute all residual variation in the observed prices to movements in discount rates, this discount rate variation would only account for 34% of variation in aggregate bond yields and 19-21% of variation in aggregate stock price ratios. Inflation expectations play a key role, driving all of the result for bond yields and substantially improving the results for stock prices relative to only using nominal earnings growth expectations. For example, for the price-earnings ratio, nominal earnings growth expectations only generate an R^2 of 54%.

Despite having constant discount rates, these fundamental prices also replicate two key return puzzles in the asset pricing literature: the predictability of stock returns and the rejection of the expectations hypothesis.[5] Empirically, movements in the price-earnings ratio

[2] These expectations may still deviate from RE. The diagnostic test, however, indicates that these deviations do not explain price variation or return puzzles.

[3] For bonds, real cash flows solely depend on inflation. For stocks, real cash flows depend on inflation as well as nominal earnings growth.

[4] For comparison, under RE, expected real cash flows and constant discount rates only explain 27%, 42%, and 40% of the variation, respectively. This is because most price movements predict future returns rather than future real cash flows (Campbell and Shiller (1988a); Ang and Piazzesi (2003); Cochrane and Piazzesi (2005); Cochrane (2011)).

[5] See Campbell and Shiller (1988a); Shiller (1979) for early summaries of these puzzles and Cochrane (2011); Gürkaynak and Wright (2012) for more recent reviews of the literature.

primarily predict future returns, with regression coefficients of -0.78 and -0.59 for ten-year nominal and real returns, respectively. Movements in the fundamental price-earnings ratio also primarily predict future returns, with nearly identical coefficients of -0.77 and -0.54.[6] Intuitively, the fundamental price-earnings ratio predicts future returns because it reflects errors in expectations of real cash flows. For bonds, we focus on excess volatility in holding returns as documented in Shiller (1979) and Singleton (1980). Under RE, the expectations hypothesis places a bound on the variance of holding returns which is violated in the data by a factor of 5.60. Holding returns based on fundamental yields violate the bound by virtually the same factor, 5.22. Even though the expectations hypothesis holds for fundamental yields, variation in errors in expectations increases the volatility of holding returns beyond what RE would imply.

To reconcile all of these findings, we propose and estimate a structural model of fundamental extrapolation which accurately replicates our results. Extrapolation is a well-studied manner of expectation formation, where agents base their expectations on a weighted sum of current and past realizations. From our decomposition of the diagnostic test, we know that beliefs about the persistence of variables and the response to surprises both play important roles in the survey data. We show that fundamental extrapolation gives a simple framework that integrates both of these elements. The model additionally nests both adaptive expectations and sticky expectations. Model agents have constant discount rates and price assets based on their biased expectations of real cash flows. We estimate all model parameters from expected and realized inflation and nominal earnings growth.

Despite not using any price information in the estimation, the model closely replicates the results of our diagnostic test both qualitatively and quantitatively. In line with our findings for observed prices, model bond and stock prices are driven by expectations of real cash flows. These prices have large comovements with forecast errors for both long-term inflation and short-term nominal earnings growth. Conversely, forecast errors for short-term inflation and long-term nominal earnings growth have virtually no comovement with prices. The stark difference in the term structure of errors is driven by the difference in the objective persistence of inflation and nominal earnings growth. For a persistent process like inflation, errors due to fundamental extrapolation are concentrated in longer horizon expectations. For a transitory process like nominal earnings growth, the errors are concentrated in short-term expectations.

Additionally, our fundamental extrapolation model is consistent with existing evidence on sticky information and accurately replicates the dynamics of survey inflation and nomi-

[6]We also get almost identical coefficients of -0.88, -0.69 and -0.92, -0.69 for the observed price-dividend ratio and the fundamental price-dividend ratio.

nal earnings growth expectations over our sample. The model nests sticky information, as the weighted sum of current and past realizations adjusts slowly over time. Coibion and Gorodnichenko (2015) find significant evidence of sticky information in survey expectations. We repeat their test on model expectations and find a sticky information coefficient of 0.65 which is consistent with their estimate of 0.54 (0.10). More generally, the model makes clear predictions about how expectations should respond to realizations. Setting model expectations equal to the 1976 survey expectations, we calculate the path of model expectations for 1976-2018 based on the realized path of inflation and nominal earnings growth. Over more than forty years, the model expectations closely track observed short-term and long-term survey expectations.

A long-standing behavioral finance literature attempts to explain return puzzles through departures from RE.[7] Our empirical analysis guides the models in this literature, informing where errors should and should not arise in expectations. Because our diagnostic test is derived from accounting identities, it applies to a broad set of models including but not limited to behavioral biases, learning, and rational inattention. We show that the relevant errors for aggregate return puzzles occur in expectations of long-term inflation and short-term nominal earnings growth, regardless of the proposed mechanism. The fact that errors differ with the forecast horizon adds to a growing literature on the term structure of subjective expectations and errors (D'Arienzo (2020); Wang (2021); Cassella et al. (2021)). Importantly, we find that the term structure of errors differs substantially depending on the variable being forecasted, similar to Afrouzi et al. (2021). Our findings can all be reconciled by a model of fundamental extrapolation, in line with previous work such as Barsky and DeLong (1993), Barberis, Shleifer, and Vishny (1998), Hirshleifer, Li, and Yu (2015), and Alti and Tetlock (2014).

This paper also adds to a large empirical literature in finance which uses survey expectations to understand aggregate asset price movements.[8] We emphasize the importance of real cash flow expectations in line with Nagel and Xu (2021). We make three key contributions to this literature. First, we provide a unified framework to study systematic errors, price variation, and return puzzles for both bonds and stocks, using inflation expectations to link the two markets. Second, we document new systematic errors in long-term inflation expectations that explain bond price movements and substantially help in explaining stock price movements relative to just using nominal earnings growth expectations. Our bond findings build on Cieslak and Povala (2015), who use a weighted average of past inflation

[7] See Hirshleifer (2015) for a survey of the literature using behavioral biases and see Collin-Dufresne, Johannes, and Lochstoer (2017) for an example using Bayesian learning.

[8] Bacchetta, Mertens, and Van Wincoop (2009); Amromin and Sharpe (2012); Greenwood and Shleifer (2014); Piazzesi, Salomao, and Schneider (2015); Giglio et al. (2021); Brunnermeier et al. (2021).

as a proxy for inflation expectations to explain yields, by highlighting that the ability of these expectations to explain yields is largely due to systematic errors. Third, we establish a test that directly determines whether subjective expectations contain deviations from RE that explain price variation and return puzzles. This builds on previous tests of deviations from RE, such as Coibion and Gorodnichenko (2015), but removes the need to assume a specific mechanism for the errors. This generalized test contributes to recent work studying aggregate nominal earnings growth expectations (De la O and Myers (2021); Bordalo et al. (2020)) by showing that only deviations from RE in short-term expectations help to explain price movements and return puzzles, while systematic errors in long-term nominal earnings growth expectations do not appear to play a role.

More broadly, a general literature has tried to understand the importance of inflation expectations. Focusing on short-term expectations and testing for different forms of errors, Ang, Bekaert, and Wei (2007), Del Negro and Eusepi (2011) and Chernov and Mueller (2012) do not find any significant errors while Coibion and Gorodnichenko (2015) find significant evidence of sticky information. We study long-term expectations in addition to short-term expectations. For short-term expectations, we find that systematic errors play no role in explaining price movements and return puzzles, in line with Cieslak (2018). For long-term expectations, we document significant predictable errors. These errors are predictable with yields for long-term bonds and stock prices, indicating that these errors influence both risk-free and risky asset prices and contribute to return puzzles. For stocks, these errors have a similar effect as the money illusion hypothesis of Modigliani and Cohn (1979), causing prices to be too low in periods of high expected inflation. However, this is not caused by investors failing to account for inflation when pricing stocks, but rather by investors accounting for inflation using biased expectations, similar to Katz, Lustig, and Nielsen (2017).

These inflation expectations are particularly relevant given the recent increase in inflation. In the 1980's, high long-term yields and high long-term inflation expectations were followed by a decline in inflation, leading to high real holding returns on long-term bonds. We now see the same pattern in current bond markets but in the opposite direction. Low long-term yields and low long-term inflation expectations over the last decade are now being followed by an increase in inflation, leading to low real holding returns on long-term bonds. Both events are consistent with our high estimate for the believed persistence of inflation and our model of fundamental extrapolation, where after a long period of high (low) inflation investors understate the chance that inflation will fall (rise).

The sections are organized as follows. Section I shows when deviation from RE explain asset price movements and return puzzles and establishes our diagnostic test. Section II describes the data, applies the diagnostic test to survey expectations at both short and long horizons, and discusses the implications for models of expectation formation. Section III measures the ability of real cash flow expectations to match observed asset prices and return puzzles. Section IV proposes a model of fundamental extrapolation in which prices are driven by biased expectations of real cash flows which matches our diagnostic test results and closely replicates the time series of survey expectations. Section V concludes.

I. Prices and Returns under Subjective Expectations

Movements in bond and stock prices must be due to changes in investors' discounted expectations of real cash flows. In this section, we establish a single necessary and sufficient condition for errors in expectations of real cash flows to explain asset price variation and return puzzles. This single condition provides a simple diagnostic test to determine if deviations from RE account for price movements and return puzzles. Importantly, not all errors are relevant, and we include an intuitive example of a systematic error that fails the diagnostic test.

We first show when deviations from RE in subjective expectations of real cash flows do and do not help to explain price movements. We then derive two additional identities which show when stock return predictability and excess volatility of bond returns can be explained by forecast errors in real cash flow expectations. For all three identities, the key condition is that forecast errors must be predictable with current prices.[9] If this condition is not satisfied, then differences between subjective expectations and RE will not explain price variation or the return puzzles.

A. *Price identities*

We start with price identities based on the definition of returns. In terms of notation, $E_t^*[\cdot]$ denotes subjective expectations, which may or may not align with standard rational expectations. For any variable x_t, we use $\tilde{x}_t$ to denote the inflation-adjusted value, $x_t - \pi_t$, where π_t is inflation.

For stocks, we use the approximate log-linearized return, which states the one-period real return in terms of real earnings growth $\Delta\tilde{e}_{t+1}$ and the price-earnings ratio pe_{t+1}, all in logs:

$$\tilde{r}_{t+1} \approx \kappa + \Delta\tilde{e}_{t+1} - pe_t + \rho pe_{t+1} \tag{1}$$

where κ is a constant, $\rho = e^{\bar{pd}}/\left(1+e^{\bar{pd}}\right) < 1$ and $\bar{pd}$ is the mean value of the log price-dividend ratio. By imposing a no-bubble condition, $\lim_{T\to\infty}\rho^T pe_{t+T} = 0$, we can iterate this

[9]Appendix A gives the full derivation of all equations in this section.

equation and apply subjective expectations to get

$$pe_t \approx \frac{1}{1-\rho}\kappa + \sum_{j=1}^{\infty} \rho^{j-1} E_t^* \left[\Delta \tilde{e}_{t+j}\right] - \sum_{j=1}^{\infty} \rho^{j-1} E_t^* \left[\tilde{r}_{t+j}\right]. \tag{2}$$

Equation (2) states that an increase in the price-earnings ratio must reflect an increase in subjective real earnings growth expectations or a decrease in subjective real return expectations. These subjective expectations do not need to be rational. Because equation (2) is derived from the definition of real returns, it holds under any probability distribution.

For bonds, we use the return from holding an n-period zero-coupon bond for one period. Expressing all variables in logs, the holding return from t to $t+1$ is defined as

$$h_{t+1}^{(n)} = ny_t^{(n)} - (n-1)\, y_{t+1}^{(n-1)}. \tag{3}$$

Iterating and applying subjective expectations, the yield is

$$y_t^{(n)} = \frac{1}{n} \sum_{j=1}^{n} E_t^* \left[\pi_{t+j}\right] + \frac{1}{n} \sum_{j=1}^{n} E_t^* \left[\tilde{h}_{t+j}^{(n+1-j)}\right] \tag{4}$$

where $\tilde{h}_t^{(n)}$ is the real holding return. Intuitively, an increase in the bond yield must be due to higher subjective inflation expectations or higher subjective real holding return expectations. This is analogous to equation (2), where prices depend on expectations of real cash flows and real returns. For bonds, real cash flows only depend on inflation, as the nominal cash flow is known, whereas for stocks this will depend on inflation and nominal earnings growth Δe_{t+j}. For the zero-coupon bond, there is a single real cash flow at maturity, so inflation in different years all have the same impact on the yield. For stocks, there are real cash flows every period, so real earnings growth in earlier years will have a larger impact on prices than real earnings growth in later years, which is reflected in the ρ^{j-1} terms.

B. *Price movements and return puzzles*

Given these equations for the price-earnings ratio and bond yields, we establish three identities which connect errors in expectations to asset prices and return puzzles. Our two return puzzles are the predictability of stock returns using price ratios, such as the price-earnings ratio or price-dividend ratio, and the rejection of the expectations hypothesis for bond yields. Other than the subjective expectations $E_t^* [\cdot]$, all operators use the objective probability distribution. For example, $Var(\cdot)$ and $Cov(\cdot, \cdot)$ denote the observable variance or covariance of variables. This allows us to use the subjective expectations observed in the survey data to explain empirical puzzles measured by an econometrician such as the comovement between current prices and future returns.

First, the comovement of prices with subjective expectations of real cash flows is

$$Cov\left(pe_t, E_t^*\left[\Delta\tilde{e}_{t+j}\right]\right) = Cov\left(pe_t, E_t\left[\Delta\tilde{e}_{t+j}\right]\right) - Cov\left(pe_t, f_t^{\Delta e_{t+j}} - f_t^{\pi_{t+j}}\right) \quad (5)$$

$$Cov\left(y_t^{(n)}, E_t^*\left[\pi_{t+j}\right]\right) = Cov\left(y_t^{(n)}, E_t\left[\pi_{t+j}\right]\right) - Cov\left(y_t^{(n)}, f_t^{\pi_{t+j}}\right) \quad (6)$$

where $E_t\left[\cdot\right]$ represents expectations under RE, $f_t^{\Delta e_{t+j}}$ is the forecast error for nominal earnings growth $\Delta e_{t+j} - E_t^*\left[\Delta e_{t+j}\right]$, and $f_t^{\pi_{t+j}}$ is the forecast error for inflation $\pi_{t+j} - E_t^*\left[\pi_{t+j}\right]$. Note that these identities hold regardless of whether subjective expectations $E_t^*\left[\cdot\right]$ are rational. If subjective expectations are not rational and the comovement of subjective expectations with prices differs from the comovement of RE expectations with prices, then there must be an observable comovement between prices and forecast errors.

Second, from equations (1)-(2), the ability of the price-earnings ratio to predict future real returns is

$$\begin{aligned} Cov\left(pe_t, \sum_{j=1}^{\infty}\rho^{j-1}\tilde{r}_{t+j}\right) &= Cov\left(pe_t, \sum_{j=1}^{\infty}\rho^{j-1}E_t^*\left[\tilde{r}_{t+j}\right]\right) \\ &+ Cov\left(pe_t, \sum_{j=1}^{\infty}\rho^{j-1}\left[f_t^{\Delta e_{t+j}} - f_t^{\pi_{t+j}}\right]\right). \end{aligned} \quad (7)$$

Note that identities (5) and (7) on the comovement of expectations with prices and the predictability of returns also hold if we replace the price-earnings ratio pe_t with the price-dividend ratio pd_t.

Third, we consider the volatility of bond holding returns and the rejection of the expectations hypothesis. The expectations hypothesis posits that expected term premia $E_t^*\left[h_{t+1}^{(n)}\right] - y_t^{(1)}$ are constant. Using equations (3)-(4), we generalize the Shiller (1979) variance bound for holding returns under the expectations hypothesis,

$$Var\left(h_{t+1}^{(n)}\right) \leq \eta Var\left(y_t^{(1)}\right) - 2nCov\left(y_t^{(n)}, \sum_{j=2}^{n} f_t^{\pi_{t+j}} + f_t^{\tilde{h}_{t+j}^{(n+1-j)}}\right) \quad (8)$$

where $\eta = n^2/\left(2n-1\right)$.

C. *Interpretation under standard rational expectations*

To provide a benchmark, we first discuss the interpretation of price movements and return puzzles under the assumption of RE. Under RE, forecast errors are uncorrelated with current prices. This means that the comovement of expectations with current prices must match the comovement of future realized real cash flows with current prices,

$$Cov\left(pe_t, E_t\left[\Delta\tilde{e}_{t+j}\right]\right) = Cov\left(pe_t, \Delta\tilde{e}_{t+j}\right) \tag{9}$$

$$Cov\left(y_t^{(n)}, E_t\left[\pi_{t+j}\right]\right) = Cov\left(y_t^{(n)}, \pi_{t+j}\right). \tag{10}$$

Empirically, the comovement of prices with future realized real cash flows is too small to explain most variation in stock and bond prices (Cochrane (2011); Ang and Piazzesi (2003)). This directly implies that under RE, expectations of real cash flows do not explain most price movements. Using the Campbell and Shiller (1988b) variance decomposition, we find over our sample that RE expectations of 10-year inflation only account for 27% of the variation in the 10-year yield.[10] While it is true that yields are highly correlated with past inflation (Cieslak and Povala (2015)), yields have a much lower comovement with future inflation. Thus, RE expectations of inflation do not account for much of the variation in yields. Similarly, RE expectations of 10-year real earnings growth only account for 42% of the variation in the price-earnings ratio.

Under RE, equations (7)-(8) provide simple tests of time-varying discount rates and the expectations hypothesis. Because forecast errors are unpredictable, the final term in equation (7) is zero. Therefore, the observed comovement between prices and future returns must be due to time-variation in investors' discount rates. Similarly, equation (8) reduces to the standard Shiller (1979) bound, which restricts the variance of holding returns to be less than $\eta Var\left(y_t^{(1)}\right)$ if the expectations hypothesis is true. Thus, under RE the observed excess volatility of holding returns beyond $\eta Var\left(y_t^{(1)}\right)$ rejects the expectations hypothesis (Singleton (1980)).

D. *Diagnostic test for deviations*

If we allow for more general subjective expectations, then equations (5)-(8) show when the results using subjective expectations will deviate from the results under RE for explaining asset prices, predictable stock returns, and excess volatility in bond holding returns. Importantly, all these phenomena share a single necessary and sufficient condition, namely that forecast errors for real cash flows must negatively comove with current prices.[11] In other words, this single condition acts as a useful diagnostic test to determine whether deviations from RE explain asset prices and return puzzles. Additionally, this allows us to measure the relative importance of subjective expectations at different horizons j by comparing how

[10]This decomposes $Var\left(y_t^{(n)}\right)$ into $\frac{1}{n}\sum_{j=1}^{n} Cov\left(y_t^{(n)}, E_t\left[\pi_{t+j}\right]\right)$ and $\frac{1}{n}\sum_{j=1}^{n} Cov\left(y_t^{(n)}, E_t\left[\tilde{h}_{t+j}^{(n+1-j)}\right]\right)$ and $Var\left(pe_t\right)$ into $\sum_{j=1}^{\infty}\rho^{j-1}Cov\left(pe_t, E_t\left[\Delta\tilde{e}_{t+j}\right]\right)$ and $\sum_{j=1}^{\infty}\rho^{j-1}Cov\left(pe_t, -E_t\left[\tilde{r}_{t+j}\right]\right)$.

[11]For stocks, the condition translates to pe_t negatively comoving with forecast errors for real earnings growth. For bonds, this condition translates to yields negatively comoving with forecast errors for inflation.

much forecast errors at different horizons comove with current prices.

If prices negatively comove with forecast errors, then equations (5)-(6) show that the comovement between subjective expectations and prices will be larger than the comovement under RE. This means that subjective expectations of real cash flows will account for more of the variation in prices than RE would imply. Even in the extreme case where prices are uncorrelated with RE expectations of real cash flows, subjective expectations of real cash flows will still comove with prices. Similarly, if the price-earnings ratio negatively comoves with forecast errors, then the price-earnings ratio will negatively predict future real returns even if discount rates $E_t^*[\tilde{r}_{t+j}]$ are constant over time.[12] Finally, forecast errors that negatively comove with the yield relax the bound in equation (8), implying that excess volatility of holding returns beyond $\eta Var\left(y_t^{(1)}\right)$ does not violate the expectations hypothesis. Predictable forecast errors generate excess volatility even if the expectations hypothesis holds, i.e., expected term premia $E_t^*\left[h_{t+1}^{(n)}\right] - y_t^{(1)}$ are constant.

In contrast, errors that do not satisfy this condition will have no impact on equations (5)-(8). To give an intuitive example of a systematic error that does not satisfy the condition, suppose there is a variable x_t that predicts real earnings growth but investors do not know this. For simplicity, assume that x_t is independent of the investors' information set I_t. In this example, an econometrician would find systematic errors in investors' expectations, as x_t predicts forecast errors. Further, the econometrician would find that x_t predicts real stock returns. However, these systematic errors do not drive price movements, as prices are only a function of I_t. As a result, these errors will not explain why prices or price ratios predict future returns.

Because this condition is derived from accounting identities, it is general and holds for any deviation from RE, e.g., learning, rational inattention, bounded rationality, and behavioral biases. This means that testing this condition is a simple way to quickly determine if systematic errors in subjective expectations help to explain price variation and return puzzles without needing to make assumptions about the primitive cause of the errors. Additionally, note that under any subjective probability distribution, errors in real return expectations equal errors in real cash flow expectations given equations (2) and (4). This means that any model of errors in real return expectations must also feature errors in real cash flow expectations, e.g., Barberis et al. (2015), Nagel and Xu (2021), and Jin and Sui (2021). Thus, this diagnostic test is still valid even in settings that focus on deviations from RE in return expectations.

[12] By extension, these forecast errors also increase the comovement between prices and nominal returns $Cov(pe_t, \tilde{r}_{t+j}) + Cov(pe_t, \pi_{t+j})$.

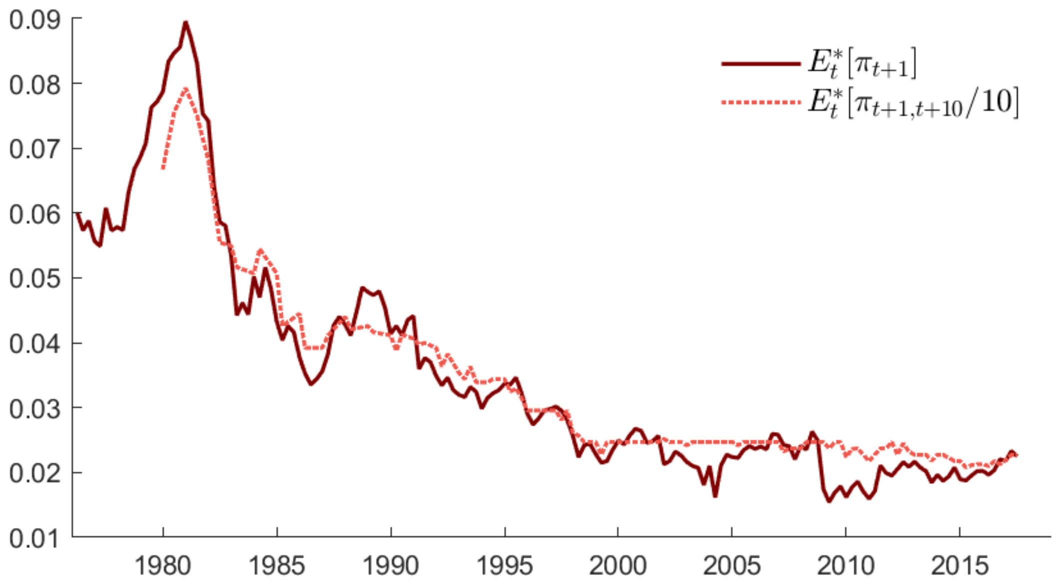

Figure 1. Subjective inflation expectations. The figure compares subjective expectations for one-year inflation (solid) and subjective expectations for annualized ten-year inflation (dotted). All variables are in logs.

II. Data and Diagnostic Test Results

In this section, we apply the diagnostic test to survey expectations of both short-term and long-term inflation and nominal earnings growth. Section II.A gives an overview of the survey data and discusses the term structure of subjective expectations. Section II.B shows the results of the diagnostic test. We find that long-term inflation expectations and short-term nominal earnings growth expectations are the only expectations that satisfy the diagnostic test from Section I for explaining asset price movements and return puzzles. Section II.C discusses the implications for models of expectation formation and further sharpens our findings by showing that the results for long-term inflation expectations are explained by the high believed persistence of inflation and the results for short-term nominal earnings growth expectations are explained by the response to surprises. Section II.D shows that the results of the diagnostic test are robust to alternative measures of survey expectations and aggregate prices, as well as a different method for determining significance.

A. Term structure of subjective expectations

A.1. Inflation Expectations

We use quarterly median inflation forecasts from the Survey of Professional Forecasters. To understand the relation between short-term and long-term inflation expectations, we analyze one-year inflation expectations from 1976Q1-2018Q2 and expectations of average inflation over the next ten years from 1979Q4-2018Q2. Figure 1 shows that one-year expectations, denoted as $E_t^*[\pi_{t+1}]$, rise sharply in the late 1970's. These expectations peak at 9% before falling steadily during the Volcker period in the 1980's and reach historic lows after 2000. Importantly, average ten-year expectations, denoted as $E_t^*[\pi_{t+1,t+10}/10]$, closely follow one-year expectation movements. At the beginning of the sample, short-term expectations are high and inflation is expected to remain high for the next ten years. At the end of the sample, we see the same pattern but in the opposite direction where short-term inflation expectations are low and inflation is expected to stay low for the next ten years. This relationship suggests that analysts believe movements in expected one-year inflation will largely persist and be observed in long-term inflation.

To quantify the relationship between expectations of short-term and long-term inflation, we measure the believed annual persistence ϕ_π^* from

$$E_t^*\left[\pi_{t+1+j}\right] \;=\; \alpha_{\pi,j} + \phi_\pi^{*j} E_t^*\left[\pi_{t+1}\right] + \varepsilon_{t,j}^{\pi}. \tag{11}$$

An increase in one-year expectations is associated with a ϕ_π^* increase in $E_t^*\left[\pi_{t+2}\right]$, ϕ_π^{*2} increase in $E_t^*\left[\pi_{t+3}\right]$, and so on. Using one-year expectations and expectations of average inflation over the next ten years $E_t^*[\pi_{t+1,t+10}/10]$, we estimate ϕ_π^* as 0.96 (0.01). The constants $\alpha_{\pi,j}$ will not matter for our analysis as they will not affect comovements or variances. Importantly, the believed persistence may differ from the objective persistence of inflation, in line with Alti and Tetlock (2014) and Afrouzi et al. (2021). In this case, long-term expectations will respond more or less to changes in short-term expectations than RE would imply. For example, a believed persistence higher than the objective persistence could cause forecasters to understate the chance of a decline (rise) in inflation after a long period of high (low) inflation, such as the decline in the 1980's or conversely the recent rise in 2020-2022.

We choose the functional form in equation (11) for three reasons. First, this form nests standard AR(1) processes, which makes the estimated value for ϕ_π^* an easily interpretable measure of how much long-term expectations move in response to a change in short-term expectations. Second, we show in Section II.B that this functional form accurately reflects the comovement of long-term expectations with asset prices, i.e., long-term inflation expectations and the approximation of long-term inflation expectations in equation (11) have nearly identical comovements with both stock and bond prices. Third, we show in Section IV that this form is consistent with a model of fundamental extrapolation which closely replicates the survey data.

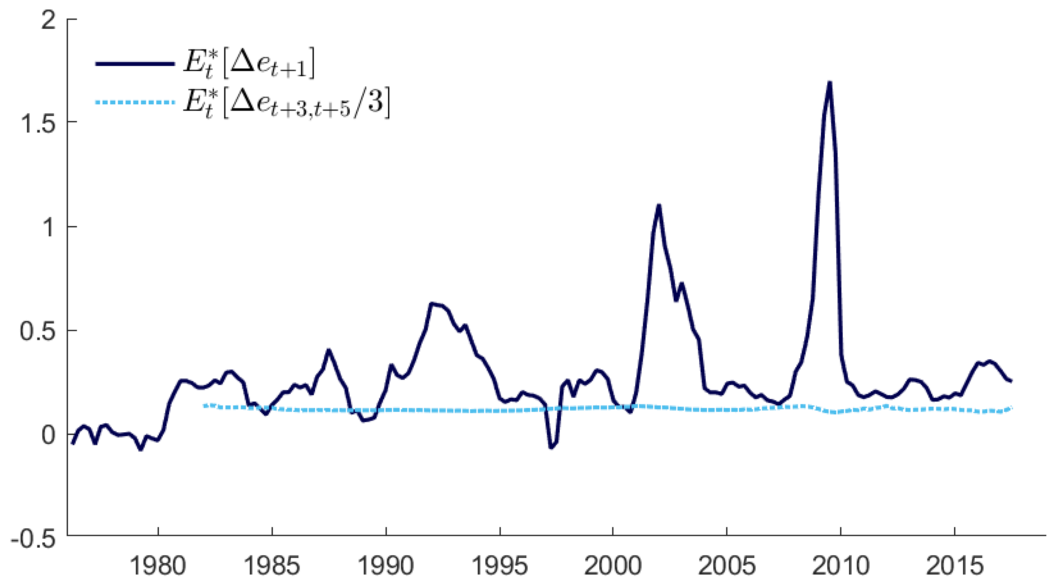

Figure 2. Subjective nominal earnings growth expectations. The figure compares subjective expectations of one-year nominal earnings growth (solid) and the subjective expectations for annualized three-to-five-year nominal earnings growth (dashed) for the S&P 500. All variables are in logs.

A.2. Nominal Earnings Growth Expectations

We construct short–term (one-year) nominal earnings growth expectations for the S&P 500 from the Thomson Reuters I/B/E/S individual firm forecasts following the methodology proposed in De la O and Myers (2021). We also estimate long-term (three-to-five year) nominal earnings growth expectations using the earnings-weighted average of long-term growth forecasts (LTG) for individual firms.[13] Figure 2 shows one-year nominal earnings growth expectations from 1976Q1 up to 2018Q2, and average three-to-five-year nominal earnings growth expectations from 1982Q1 to 2018Q2. Expectations of one-year nominal earnings growth are volatile, with a standard deviation of 26.4%, rising substantially after the large decline in earnings during financial crisis. In comparison, expectations of three-to-five-year nominal earnings growth have much lower volatility, with a standard deviation of only 1.2%. For robustness, we show in Section II.D that our results are unchanged if we use the value-weighted average of long-term growth forecasts from Bordalo et al. (2020) which have a similarly low volatility of only 1.4%.

The high volatility of short-term expectations relative to long-term expectations suggests that movements in nominal earnings growth expectations are primarily concentrated at short horizons. The fact that movements in short-term expectations are not matched by similar movements in long-term expectations indicates that analysts believe movements in expected

[13] Full details on the construction can be found in Appendix B.

one-nominal earnings growth will not persist through to long-term nominal earnings growth. We quantify the relationship between expectations of short-term and long-term nominal earnings growth by measuring the believed annual persistence ϕ_e^* from

$$E_t^*[\Delta e_{t+j}] \;=\; \alpha_{e,j} + \phi_e^{*j} E_t^*[\Delta e_{t+1}] + \varepsilon_{t,j}^e. \tag{12}$$

Using the one-year and three-to-five-year expectations, we estimate a small ϕ_e^* of 0.004 (0.075).[14] This sharply contrasts with the large, significant estimate of ϕ_π^* of 0.96. Just as with inflation expectations, we show in Section II.B that this estimate for ϕ_e^* accurately capture the comovement of long-term nominal earnings growth expectations with stock price ratios and the functional form of equation (12) is consistent with the fundamental extrapolation model of Section IV.

A.3. Real earnings growth expectations

Throughout the paper, we split real earnings growth into nominal earnings growth and inflation to align with the available survey data. Looking at Figures 1 and 2, we see that short-term real earnings growth expectations are primarily driven by changes in short-term nominal earnings growth expectations, with short-term inflation expectations playing only a small part. However, inflation expectations do play an important role at long horizons. While long-term nominal earnings growth expectations have virtually no trend over time, long-term inflation expectations have dropped considerably. This implies that expectations of real earnings growth at long horizons have substantially increased over time.

One might expect that the lack of a decline in long-term nominal earnings growth expectations must represent a systematic error. However, over this period, realized nominal earnings growth also did not decline despite the large drop in realized inflation. Section II.B formally shows that forecast errors in expectations of long-term nominal earnings growth do not play a role in explaining asset prices. Instead, Section II.B finds that forecast errors in long-term inflation expectations explain asset prices, indicating that the relevant errors in long-term real earnings growth expectations come from inflation expectations.

B. Diagnostic test results

Section I establishes a diagnostic test for deviations from RE in subjective expectations of real cash flows to explain asset prices, stock return predictability, and excess bond return volatility. The necessary and sufficient condition is that current prices must predict forecast

[14]As with the estimation of the believed persistence of inflation, the constants $\alpha_{e,j}$ will not matter for our analysis as they will not affect comovements or variances.

Table I

Diagnostic test for inflation and nominal earnings growth expectations

This table shows the results of the diagnostic test which determines if differences between subjective expectations and RE explain asset price movements and return puzzles. The condition is that prices must comove with forecast errors. In *Panel A*, the first row shows the covariance of the S&P 500 price-earnings ratio with short-term inflation expectations $E_t^*[\pi_{t+1}]$, realized short-term inflation π_{t+1}, and the forecast errors $\pi_{t+1} - E_t^*[\pi_{t+1}]$ from 1976Q1 to 2018Q2. The second row shows the covariance of the S&P 500 price-earnings ratio with long-term inflation expectations $E_t^*[\pi_{t+1,t+10}]$, realized long-term inflation $\pi_{t+1,t+10}$, and the forecast errors $\pi_{t+1,t+10} - E_t^*[\pi_{t+1,t+10}]$ using quarterly data from 1979Q4 to 2018Q2. The third and fourth rows show analogous results using the ten-year Treasury yield instead of the S&P 500 price-earnings ratio. In *Panel B*, the first row shows the covariance of the S&P 500 price-earnings ratio with short-term nominal earnings growth expectations $E_t^*[\Delta e_{t+1}]$, realized short-term nominal earnings growth Δe_{t+1}, and the forecast errors $\Delta e_{t+1} - E_t^*[\Delta e_{t+1}]$ from 1976Q1 to 2018Q2. The second row shows the covariances of the S&P 500 price-earnings ratio with long-term nominal earnings growth expectations $E_t^*[\Delta e_{t+3,t+5}]$, realized long-term nominal earnings growth $\Delta e_{t+3,t+5}$ and the forecast errors $\Delta e_{t+3,t+5} - E_t^*[\Delta e_{t+3,t+5}]$ from 1982Q1 to 2018Q2. Expectations expressed in percentages. We use small-sample adjusted Newey-West standard errors. Superscripts indicate significance at the 1% (***), 5% (**), and 10% (*) level.

	Horizon	Expected	Realized	Forecast Error
Panel A: Inflation				
$Cov(pe_t, \cdot)$	Short-term (π_{t+1})	-0.66^{***}	-0.82^{***}	-0.16
	Long-term ($\pi_{t+1,t+10}$)	-3.97^{**}	-1.66^{*}	2.31^{***}
$Cov\left(y_t^{(10)}, \cdot\right)$	Short-term (π_{t+1})	0.04^{***}	0.04^{***}	0.00
	Long-term ($\pi_{t+1,t+10}$)	0.25^{***}	0.13^{***}	-0.13^{***}
Panel B. Nominal Earnings Growth				
$Cov(pe_t, \cdot)$	Short-term (Δe_{t+1})	9.44^{***}	5.18	-4.26^{***}
	Long-term ($\Delta e_{t+3,t+5}$)	0.36	2.71	2.35

errors for real cash flows. For stocks, this means that the current price-earnings ratio positively predicts forecast errors for inflation and negatively predicts forecast errors for nominal earnings growth. For bonds, this means that the current yield negatively predicts forecast errors for inflation. In this section, we empirically test if subjective expectations satisfy this condition. We find that expectations of long-term inflation and short-term nominal earnings growth pass the test while expectations of short-term inflation and long-term nominal earnings growth do not. Table I shows the results.

We first focus on short-term inflation expectations, which have been shown to significantly deviate from RE based on tests of sticky information (Coibion and Gorodnichenko (2015)). Despite this deviation from RE, we find that forecast errors for short-term inflation expectations do not satisfy the diagnostic test for the price-earnings ratio. The first row of Panel A shows the comovement of the price-earnings ratio with expected and realized short-term inflation, as well as the comovement with the forecast error. We see that expected short-term inflation and realized short-term inflation both have significant negative comovements with the price-earnings ratio and that the magnitudes are quite similar. As a result, forecast errors do not significantly comove with the price-earnings ratio, meaning that forecast errors for short-term inflation do not explain movements in the price-earnings ratio or the stock return puzzle.

In contrast, we document new systematic errors in long-term inflation expectations that do satisfy the diagnostic test. Consistent with the findings of Section II.A, we find a large amount of action in long-term inflation expectations. The second row of Panel A shows that long-term inflation expectations have a significant negative comovement with the price-earnings ratio of -3.97 that is substantially larger than the comovement of short-term expectations with the price-earnings ratio of -0.66. This is also significantly larger than the covariance of the price-earnings ratio with future realized long-term inflation of -1.66, leading to significant forecast errors that are predictable with the current price-earnings ratio. This matches our high believed persistence ϕ_π^* of 0.96, as $E_t^*\left[\pi_{t+1}\right]\left(1-\phi_\pi^{*10}\right)/\left(1-\phi_\pi^*\right)$ has a nearly identical covariance with the price-earnings ratio of -3.85 over the same sample.

We find the same results when we test the condition using bond yields rather than the price-earnings ratio, meaning that systematic errors in long-term inflation expectations also explain price movements and return puzzles for bonds, while systematic errors in short-term inflation do not. Expected and realized short-term inflation have virtually the same comovement with the ten-year bond yield at 0.04, leading to an insignificant comovement between the yield and forecast errors. Compared to short-term expectations, long-term expectations comove much more with the yield at 0.25, again emphasizing the importance of movements in longer horizon inflation expectations. This comovement is substantially larger than the comovement of future realized long-term inflation with the yield of 0.13, meaning that forecast errors significantly comove with the yield. Again, this matches our high persistence estimate ϕ_π^* of 0.96, as $E_t^*\left[\pi_{t+1}\right]\left(1-\phi_\pi^{*10}\right)/\left(1-\phi_\pi^*\right)$ has a covariance of 0.23 with the yield.

Next, we test expectations of nominal earnings growth and show that forecast errors for short-term expectations satisfy the diagnostic test. The first row of Panel B shows that short-term nominal earnings growth expectations have a large and significant comovement with the price-earnings ratio, while realized short-term nominal earnings growth has a smaller and insignificant comovement. This means that forecast errors significantly comove with the price-earnings ratio.

Finally, we show that forecast errors for long-term nominal earnings growth fail the diagnostic test. In contrast with short-term expectations, long-term nominal earnings growth expectations have a small and insignificant comovement with the price-earnings ratio of 0.36. Consistent with our low estimate for the believed persistence ϕ_e^* of 0.004, this covariance is nearly two orders of magnitude smaller than the covariance of short-term nominal earnings growth expectations with the price-earnings ratio of 9.44. Importantly, realized long-term nominal earnings growth is also insignificantly related to the price-earnings ratio, meaning that forecast errors do not significantly comove with the price-earnings ratio. While expectations of long-term nominal earnings growth may fail other tests of RE, as shown by Bordalo et al. (2020), the inability of the price-earnings ratio to predict the forecast errors implies that these deviations from RE will not help in matching prices or explaining return puzzles. In fact, the lack of comovement between long-term nominal earnings growth expectations and the price-earnings ratio slightly worsens the ability to explain prices and return puzzles, as expected long-term nominal earnings growth comoves less with the price-earnings ratio than realized long-term nominal earnings growth.

It is important to note that our results still imply that there are significant errors in expected long-term nominal earnings that are relevant for price variation and return puzzles. Holding growth fixed for all following years, a 10-percentage point increase in one-year nominal earnings growth raises the level of all future nominal earnings by 10%. Thus, mistakes about one-year nominal earnings growth generates mistakes in all expected future nominal earnings. This is why short-term nominal earnings growth expectations can have a large impact on prices. Mistakes about long-term nominal earnings growth would generate additional errors solely in expected long-term nominal earnings but not in expected short-term nominal earnings. Our findings show that this latter type of mistake is not correlated with prices. However, mistakes about long-term nominal earnings will still be relevant via the errors in short-term nominal earnings growth expectations.

C. Implications for models of expectation formation

The results of Table I provide several insights for models of deviations from RE that attempt to explain asset prices. First, our results apply to both stock and bond markets which points against models where agents make errors in one market but not the other, such as models of money illusion.[15] Second, the diagnostic test reveals that price variation and return puzzles are driven by errors in expectations of long-term inflation and short-term nominal earnings growth, while errors in expectations of short-term inflation and long-term nominal

[15] These models predict that inflation expectations lead to mispricing for stocks but not for bonds. See Modigliani and Cohn (1979) and more recently Cohen, Polk, and Vuolteenaho (2005).

earnings growth do not appear to play a role. Thus, the relevant errors differ across horizons and the term structure differs depending on the variable being forecasted. Third, for both long-term inflation and short-term nominal earnings growth, these errors arise because the comovement of subjective expectations with prices is substantially larger in magnitude than the value that RE would imply. Section IV shows that these results are all consistent with a model of fundamental extrapolation.

In this section, we further sharpen our findings by showing that the large comovement of long-term inflation expectations and short-term nominal earnings growth expectations with prices is mainly explained by the high believed persistence of inflation and the response of short-term nominal earnings growth expectations to surprises. Because the diagnostic test is based on comovements, we can decompose these comovements to better understand the implications for models of expectation formation. In other words, once the diagnostic test has identified which subjective expectations help to explain price variation and return puzzles, we can investigate these specific comovements further to shed more light on possible mechanisms.

For long-term inflation expectations, we find that the high comovement with prices is almost entirely explained by the high believed persistence ϕ_π^*. We can express long-term inflation expectations as

$$E_t^*\left[\pi_{t+1,t+10}\right] = \alpha_\pi^{LT} + \frac{1-\phi_\pi^{*10}}{1-\phi_\pi^*} E_t^*\left[\pi_{t+1}\right] + \varepsilon_{t,\pi}^{LT} \tag{13}$$

where $\frac{1-\phi_\pi^{*10}}{1-\phi_\pi^*} E_t^*\left[\pi_{t+1}\right]$ represents movements in long-term expectations due to the believed persistence and $\varepsilon_{t,\pi}^{LT}$ represents all other movements in long-term inflation expectations. By extension, the comovement of prices with $E_t^*\left[\pi_{t+1,t+10}\right]$ can be decomposed into the comovement of prices with $\frac{1-\phi_\pi^{*10}}{1-\phi_\pi^*} E_t^*\left[\pi_{t+1}\right]$ and the comovement of prices with $\varepsilon_{t,\pi}^{LT}$. As mentioned in Section II.B and as formally shown in Table II Panel A, the large comovement between long-term inflation expectations and the current prices is almost entirely attributed to $\frac{1-\phi_\pi^{*10}}{1-\phi_\pi^*} E_t^*\left[\pi_{t+1}\right]$. As a result, shocks or information that only impact long-term inflation expectations $\varepsilon_{\pi,t}^{LT}$ are not important for explaining this large comovement.

For short-term nominal earnings growth expectations, we find that the large comovement of subjective expectations with prices is largely explained by the response of short-term expectations to surprises in nominal earnings growth. In line with the earnings growth reversal model of De la O and Myers (2021), we separate movements in short-term nominal earnings growth expectations into a response to the recent earnings growth surprise $\beta_e\left(E_{t-1}^*\left[\Delta e_t\right] - \Delta e_t\right)$ and all other movements $\varepsilon_{t,e}^{ST}$,

$$E_t^*\left[\Delta e_{t+1}\right] = \alpha_e^{ST} + \beta_e\left(E_{t-1}^*\left[\Delta e_t\right] - \Delta e_t\right) + \varepsilon_{t,e}^{ST}. \tag{14}$$

Table II

Decomposing comovement with price ratios

This table decomposes the result of the diagnostic test for long-term inflation expectations and short-term nominal earnings growth expectations. It shows the covariance of prices and yields with each element in equations (13) and (14). In *Panel A*, the first row shows the covariance of the S&P 500 price-earnings ratio with long-term inflation expectations $E_t^*\left[\pi_{t+1,t+10}\right]$, and with each of its two components, $\frac{1-\phi_\pi^{*10}}{1-\phi_\pi^*}E_t^*\left[\pi_{t+1}\right]$ and $\varepsilon_{t,\pi}^{LT}$. The second row show analogous results using the ten-year Treasury yield instead of the S&P 500 price-earnings ratio. *Panel B* shows the covariance of the S&P 500 price-earnings ratio with short-term nominal earnings growth expectations $E_t^*\left[\Delta e_{t+1}\right]$ and with each of its two components, $\beta_e\left(E_{t-1}^*\left[\Delta e_t\right]-\Delta e_t\right)$ and $\varepsilon_{t,e}^{ST}$. Expectations expressed in percentages. We use small-sample adjusted Newey-West standard errors. Superscripts indicate significance at the 1% (***), 5% (**), and 10% (*) level.

Panel A: Inflation			
	$E_t^*\left[\pi_{t+1,t+10}\right]$	$\frac{1-\phi_\pi^{*10}}{1-\phi_\pi^*}E_t^*\left[\pi_{t+1}\right]$	$\varepsilon_{t,\pi}^{LT}$
$Cov\left(pe_t,\cdot\right)$	-3.97^{***}	-3.85^{***}	-0.12
$Cov\left(y_t^{(10)},\cdot\right)$	0.25^{***}	0.23^{***}	0.02
Panel B. Nominal Earnings Growth			
	$E_t^*\left[\Delta e_{t+1}\right]$	$\beta_e\left(E_{t-1}^*\left[\Delta e_t\right]-\Delta e_t\right)$	$\varepsilon_{t,e}^{ST}$
$Cov\left(pe_t,\cdot\right)$	9.31^{***}	8.46^{***}	0.88^{***}

Estimating equation (14) gives a β_e of 0.74 (0.02). As shown in Table II Panel B, the large comovement between short-term nominal earnings growth expectations and the current prices is almost entirely attributed to the response to the recent surprise $\beta_e\left(E_{t-1}^*\left[\Delta e_t\right]-\Delta e_t\right)$. Section IV shows that a model of fundamental extrapolation can match both this fact and our finding that the large comovement of long-term inflation expectations with prices is mainly explained by a high believed persistence.

D. Robustness checks

In this section, we confirm that our results are robust to alternative measures of survey expectations, aggregate prices, and methods for determining significance. Using the long-term nominal earnings growth expectations of Bordalo et al. (2020) and consumer inflation expectations from the Michigan survey produces virtually identical results for the diagnostic test as Table I. Similarly, our results are robust to using the price-dividend ratio rather than the price-earnings ratio. Finally, we account for overlapping observations in long-term

Table III

Value-weighted long-term nominal earnings growth expectations

This table compares the earnings-weighted long-term nominal earnings growth expectation to the value-weighted long-term nominal earnings growth expectation. The first row compares the standard deviation of the series. The second row reports the believed persistence estimated from equation (12) using the short-term and long-term nominal earnings growth expectations. The third and fourth rows show the covariance of the S&P 500 price-earnings ratio with the long-term nominal earnings growth expectations $E_t^* [\Delta e_{t+3,t+5}]$ and the forecast errors $\Delta e_{t+3,t+5} - E_t^* [\Delta e_{t+3,t+5}]$. All estimates are calculated using a sample of 1982Q1 to 2018Q2. Small-sample adjusted Newey-West standard errors reported in parenthesis.

	Earnings-weighted	Value-weighted
Standard deviation	1.2%	1.4%
Believed persistence ϕ_e^*	0.004	0.008
	(0.075)	(0.049)
Comovement with pe_t	0.36	0.55
	(0.40)	(0.48)
Comovement of forecast	2.35	2.16
errors with pe_t	(1.65)	(1.57)

inflation expectations using the methodology Bauer and Hamilton (2018) and show that our findings are still significant even under the worst-case scenario.

The low volatility of long-term nominal earnings growth expectations and their low comovement with the price-earnings ratio may be surprising given the high volatility and comovement with prices for short-term nominal earnings growth expectations. For robustness, we repeat our analysis using the value-weighted long-term nominal growth expectations of Bordalo et al. (2020).[16] For any set of firms, such as the S&P 500, the growth of total earnings is equal to the earnings-weighted average of individual firm earnings growth. Therefore, we use the earnings-weighted average for our main analysis. However, we find that using the value-weighted average of long-term nominal earnings growth expectations does not change our results in any noticeable way. Specifically, we continue to find that long-term nominal earnings growth expectations (i) have low volatility compared to short-term nominal earnings growth expectations, (ii) imply that the believed persistence of nominal earnings growth is small, and most importantly (iii) fail the diagnostic test.

Table III shows the results. The standard deviations for the earnings-weighted and value-weighted expectations closely align at 1.2% and 1.4%, respectively, and are both substantially lower than the 26.4% standard deviation of short-term nominal earnings growth expectations. A natural consequence of this is that the believed persistence is nearly 0 as shown in the second row. The third row shows that the comovement of value-weighted expectations

[16]We thank Andrei Shleifer and Rafael La Porta for sharing these expectations.

Table IV

Diagnostic test with price-dividend ratio

This table shows the results of the diagnostic test using price-dividend ratio instead of price-earnings ratio. The condition is that prices must comove with forecast errors. In *Panel A*, the first row shows the covariance of the S&P 500 price-dividend ratio with short-term inflation expectations $E_t^*[\pi_{t+1}]$, realized short-term inflation π_{t+1}, and the forecast errors $\pi_{t+1} - E_t^*[\pi_{t+1}]$ from 1976Q1 to 2018Q2. The second row shows the covariance of the S&P 500 price-dividend ratio with long-term inflation expectations $E_t^*[\pi_{t+1,t+10}]$, realized long-term inflation $\pi_{t+1,t+10}$, and the forecast errors $\pi_{t+1,t+10} - E_t^*[\pi_{t+1,t+10}]$ using quarterly data from 1979Q4 to 2018Q2. In *Panel B*, the first row shows the covariance of the S&P 500 price-dividend ratio with short-term nominal earnings growth expectations $E_t^*[\Delta e_{t+1}]$, realized short-term nominal earnings growth Δe_{t+1}, and the forecast errors $\Delta e_{t+1} - E_t^*[\Delta e_{t+1}]$ from 1976Q1 to 2018Q2. The second row shows the covariances of the S&P 500 price-dividend ratio with long-term nominal earnings growth expectations $E_t^*[\Delta e_{t+3,t+5}]$, realized long-term nominal earnings growth $\Delta e_{t+3,t+5}$ and the forecast errors $\Delta e_{t+3,t+5} - E_t^*[\Delta e_{t+3,t+5}]$ from 1982Q1 to 2018Q2. Expectations expressed in percentages. We use small-sample adjusted Newey-West standard errors. Superscripts indicate significance at the 1% (***), 5% (**), and 10% (*) level.

	Horizon	Expected	Realized	Forecast Error
Panel A: Inflation				
$Cov(pd_t, \cdot)$	Short-term (π_{t+1})	-0.70^{***}	-0.77^{***}	-0.08
	Long-term ($\pi_{t+1,t+10}$)	-5.13^{***}	-2.25^{***}	2.88^{***}
Panel B. Nominal Earnings Growth				
$Cov(pd_t, \cdot)$	Short-term (Δe_{t+1})	2.52	-0.90	-3.43^{**}
	Long-term ($\Delta e_{t+3,t+5}$)	0.59	0.84	0.25

with the price-earnings ratio is small and insignificant at 0.55 (0.48), matching our findings using the earnings-weighted expectations of 0.36 (0.40). Crucially, forecast errors for both the value-weighted expectations and the earnings-weighted expectations have no significant comovement with the price-earnings ratio, as shown in the fourth row.

Similarly, our inflation results are unchanged if we use short-term and long-term inflation expectations from the Michigan Survey of Consumers, rather than the Survey of Professional Forecasters. As shown in Table AI in Appendix C.1, the comovement of short-term forecast errors with prices is small and insignificant, while the comovement of long-term forecast errors with prices is large and significant. The results are not only qualitatively similar but also quantitatively match the values of Table I. Further, the believed persistence measured from these surveys is high at 0.95 (0.02) and virtually identical to the value measured from the Survey of Professional Forecasters of 0.96 (0.01). These expectations also closely match the volatility of short-term and long-term inflation expectations from the Survey of Professional Forecasters.

As an additional measure of robustness, we repeat the diagnostic test of Table I using the price-dividend ratio rather than the price-earnings ratio to show that the choice of the normalizing variable does not change the results. In line with the results for the price-earnings ratio, Table IV shows that forecast errors for long-term inflation and short-term nominal earnings growth significantly comove with the price-dividend ratio. Further, the magnitude of these comovements closely matches the results using the price-earnings ratio. For long-term inflation forecast errors, the comovement with the price-dividend ratio is 2.88 and the comovement with the price-earnings ratio is 2.31. Similarly, for short-term nominal earnings growth forecast errors, the comovement with the price-dividend ratio and price-earnings ratio is -3.43 and -4.26, respectively. Forecast errors for short-term inflation and long-term nominal earnings growth do not significantly comove with the price-dividend ratio, matching the results for the price-earnings ratio.

To ensure that the significance of our inflation results is not due to persistent variables or overlapping observations in long-term inflation expectations, we utilize the methodology of Bauer and Hamilton (2018). As detailed in Appendix C.2, even after accounting for autocorrelation in the variables and assuming the worst-case scenario where forecast errors perfectly overlap over time (i.e. $\pi_{t+j} - E_t^* [\pi_{t+j}] = \pi_{t+j} - E_{t+k}^* [\pi_{t+j}]$ for all $k < j$), the p-value for the covariance of the price-earnings ratio with long-term inflation forecast errors is only 0.015. Similarly, the p-value is only 0.014 using yields rather than the price-earnings ratio and is 0.005 using the price-dividend ratio.[17]

III. Asset Prices and Return Puzzles

In this section, we show that subjective expectations of inflation and nominal earnings growth accurately match observed asset prices and return puzzles. As discussed in Section II, this is due to errors in long-term inflation expectations and short-term nominal earnings growth expectations.

We construct fundamental stock and bond prices based on equations (2) and (4) using subjective expectations of real cash flows and constant discount rates. We focus on aggregate stock and bond prices, specifically the S&P 500 price-earnings ratio and price-dividend ratio and the ten-year zero-coupon Treasury yield. Fundamental prices closely match all three time-series. The quantitative results imply that time-varying discount rates play almost no role in aggregate stock prices and play only a secondary role in aggregate bond yields.

[17]This worst-case scenario substantially overstates the likelihood of spuriously finding a large comovement as it assumes that forecast errors are much more persistent than what we observe in the data. Thus, these p-values are an upper bound.

We then test whether these fundamental prices replicate two key puzzles in the asset pricing literature on stocks and bonds: the predictability of stock returns and the rejection of the expectations hypothesis for bonds. Specifically, we study the comovement of future stock returns with current price ratios and the excess volatility of bond holding returns. Under RE, the magnitude of these two puzzles is evidence that discount rates must vary over time and that this variation is substantial. In both cases, the fundamental prices closely match the observed puzzles despite having constant discount rates.

A. *Price movements*

Movements in observed asset prices are closely matched by subjective expectations of real cash flows. To construct a fundamental price-earnings ratio, we first set the discount rate to a constant. Second, we use the believed persistence for inflation and nominal earnings growth ϕ_π^* and ϕ_e^*, which are shown in Section II.B to accurately capture the comovement of long-term inflation and nominal earnings growth expectations with the price-earnings ratio. The fundamental price-earnings ratio is then

$$pe_t^{fun} = c + \frac{1}{1-\rho\phi_e^*} E_t^*\left[\Delta e_{t+1}\right] - \frac{1}{1-\rho\phi_\pi^*} E_t^*\left[\pi_{t+1}\right] \quad (15)$$

where the discount rate simply falls into the constant c.[18]

Note that the fundamental price-earnings ratio is simply a linear function of two time-series measured from survey forecasts, $E_t^*\left[\Delta e_{t+1}\right]$ and $E_t^*\left[\pi_{t+1}\right]$, and the parameters ϕ_e^* and ϕ_π^* are also estimated directly from the survey data.[19] The mean price-earnings ratio is used to set c, but no other information about the observed price-earnings ratio is used when constructing the fundamental price-earnings ratio. By extension, the fundamental price-dividend ratio is

$$pd_t^{fun} = pe_t^{fun} + e_t - d_t. \quad (16)$$

Similarly, using a constant discount rate, the fundamental ten-year yield is

$$y_t^{(10),fun} = c^y + \frac{1}{10}\frac{1-\phi_\pi^{*10}}{1-\phi_\pi^*} E_t^*\left[\pi_{t+1}\right] \quad (17)$$

where the second term represents the average expectation of inflation over the next ten years. The results are almost identical if we use the survey ten-year inflation expectations $E_t^*\left[\pi_{t+1,t+10}\right]$. This is because the correlation between one-year and ten-year inflation expec-

[18]The term c conveniently condenses all of the constants. Given the constant discount rate $\tilde{r}$, the full expressions is $c = \frac{\kappa-\tilde{r}}{1-\rho} + \sum_{j=1}^{\infty} \rho^{j-1}\left(\alpha_{e,j} - \alpha_{\pi,j}\right)$.

[19]In Appendix C.3, we also estimate a generalization of equations (11) and (12) where expectations of inflation can impact expectations of nominal earnings growth. When estimated on the survey expectations, we find no significant interaction and all of our results are robust to including this interaction.

Table V

Explaining price movements

This table shows a comparison between observed stock and bond prices and the fundamental prices constructed with the subjective expectations of inflation and nominal earnings growth. The table shows linear regressions of observed prices on fundamental prices and reports the intercept a, the slope b, and the R^2 of the regression. Additionally, we report the constrained R^2 obtained from the residuals of the constrained regression where the intercept is forced to be zero $a = 0$ and the slope is forced to be one $b = 1$. The first two rows show the results for the S&P 500 price-earnings ratio and price-dividend ratio. The third row shows the results for the 10-year Treasury yield. The fourth and fifth row show the results for the long-run component pe_t^{LR} and the short-run component pe_t^{SR} of the S&P 500 price-earnings ratio. The short-run and long-run component were obtained by applying a Hodrick-Prescott filter on both the observed and the fundamental price-earnings ratio with a smoothing parameter of 1600. All calculations use quarterly data. Small-sample adjusted Newey-West standard errors in parenthesis.

Regression	a	b	R^2	Constrained R^2 ($a=0, b=1$)
$pe_t = a + bpe_t^{fun} + \varepsilon_t$	0.13	0.96	0.81	0.81
	(0.21)	(0.09)		
$pd_t = a + bpd_t^{fun} + \varepsilon_t$	0.16	0.96	0.79	0.79
	(0.39)	(0.12)		
$y_t = a + by_t^{fun} + \varepsilon_t$	-0.03	1.55	0.66	0.58
	(0.02)	(0.30)		
$pe_t^{LR} = a + bpe_t^{fun,LR} + \varepsilon_t$	0.03	0.99	0.82	0.82
	(0.14)	(0.06)		
$pe_t^{SR} = a + bpe_t^{fun,SR} + \varepsilon_t$	0.00	0.90	0.81	0.80
	(0.01)	(0.05)		

tations is 0.98 and ϕ_π^* is estimated to match the ten-year expectations. The constant c^y is set to match the mean ten-year yield, but no other information about the observed yield is used in the fundamental yield.

Table V shows regressions of the observed stock price ratios and bond yield on the fundamental price ratios and bond yield. In short, subjective expectations of real cash flows explain virtually all movements in stock price ratios and 2/3 of movements in bond yields. For the price-earnings ratio and the price-dividend ratio, the observed values move almost 1-1 with the fundamental values with coefficients of 0.96 in both regressions and high R^2's of 0.81 and 0.79, respectively. Even for bond yields, which one may expect to be strongly driven by discount rates, we find a significant coefficient of 1.55 and an R^2 of 0.66. Additionally, we cannot reject that the observed prices and yields are equal to the fundamental prices and yields plus noise, i.e., $a = 0$ and $b = 1$. In this constrained case of $a = 0$ and $b = 1$, we continue to find high R^2's of roughly 0.8 for the stock price ratios and 0.58 for the bond yield.

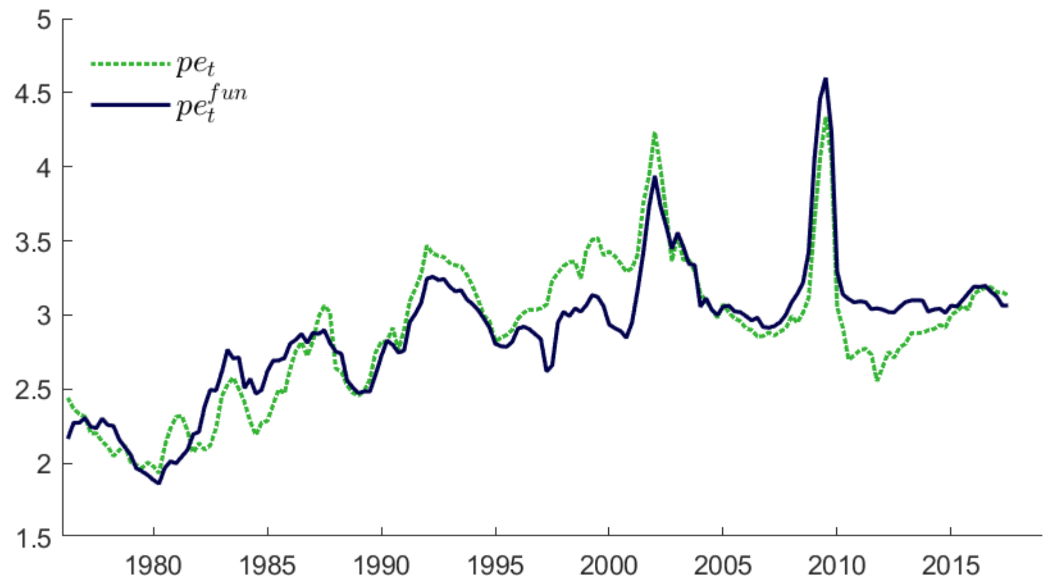

Figure 3. Fundamental and observed price-earnings ratio. The figure compares the observed price-earnings ratio for the S&P 500 (dotted green) with the fundamental price-earnings ratio (solid blue). The fundamental price-earnings ratio is the value of subjective expectations of real earnings growth plus a constant. All variables are in logs.

The results of Table V leave little room for large variation in discount rates. The high R^2 values demonstrate that the majority of price movements for both stocks and bonds are attributed to movements in subjective expectations of real cash flows. Even if we ascribe all of the residual variation to movements in discount rates, the discount rate variation would play almost no role in stock prices and only a secondary role in bond yields. Inflation expectations play a particularly important role as they account for the entirety of the bond results and substantially improve the stock results relative to only using nominal earnings growth expectations. To measure this, we re-estimate the first row of Table V holding inflation expectations constant and only find an unconstrained R^2 of 0.54.[20]

To emphasize how closely subjective expectations of real cash flows match observed stock prices, we calculate two additional measures and test how well the fundamental prices explain long-run changes in the price-earnings ratio as well as short-run business cycle fluctuations. Using a standard Hodrick-Prescott filter, we calculate the long-run and short-run components of the observed price-earnings ratio, pe_t^{LR} and pe_t^{SR}, and the fundamental price-earnings ratio, $pe_t^{fun,LR}$ and $pe_t^{fun,SR}$. The final two rows of Table V show that the fundamental values continue to move almost 1-1 with the observed values, with coefficients of 0.99 and 0.90 and high R^2's of 0.82 and 0.81 for the long-run and short-run components, respectively.

[20]Including long-term nominal earnings growth expectations as a separate independent variable in the regression only marginally improves the unconstrained R^2 to 0.57.

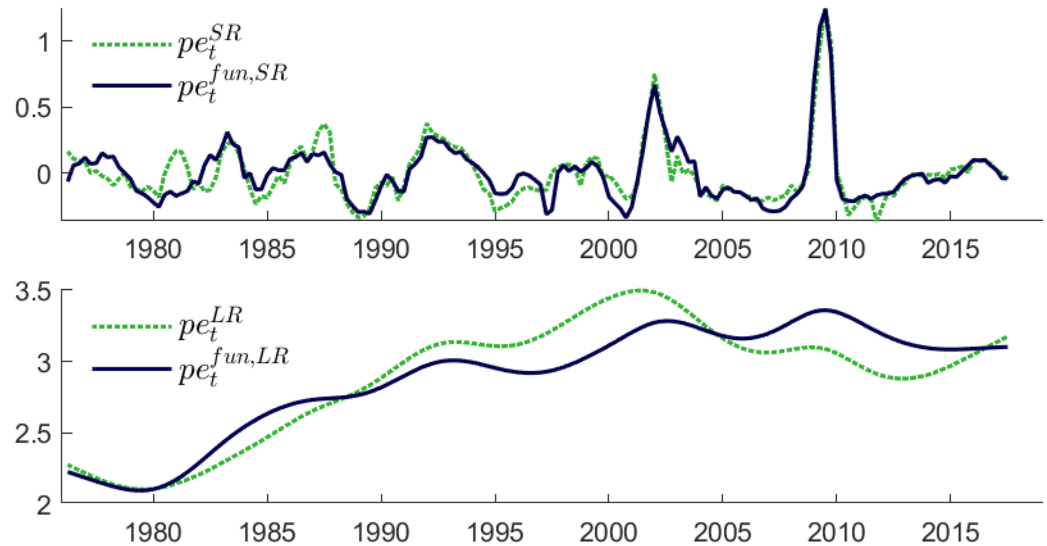

Figure 4. Short-run and long-run price movements. The top panel compares the short-run component of the price-earnings ratio for the S&P 500 (dotted green) with the short-run component of the fundamental price-earnings ratio (solid blue). The fundamental price-earnings ratio is the value of subjective expectations of real earnings growth plus a constant. The bottom panel compares the long-run component of the price-earnings ratio and the long-run component of the fundamental price-earnings ratio. All variables are in logs.

Figures 3 and 4 show the fundamental and observed price-earnings ratio, as well as the short-run and long-run components. Figure 3 shows that the fundamental price-earnings ratio pe_t^{fun} accurately tracks the observed price-earnings ratio pe_t. Additionally, Figure 4 shows that the fundamental price-earnings ratio captures almost all short-run business-cycle variation in the price-earnings ratio, as well as the long-run secular increase in the price-earnings ratio over the last 40 years.

B. Stock return predictability

A central argument for time-varying discount rates is the empirical fact that future stock returns comove significantly with current price ratios. As shown in equation (7), under RE this comovement must be due to substantial variation in discount rates. In Table VI, we test whether this comovement is matched by fundamental price ratios. Intuitively, if predictable returns are driven by errors in subjective expectations of real cash flows, then fundamental prices with constant discount rates will also predict returns.

The first row of Table VI shows the results of univariate regressions of future nominal returns for the next ten years on current price ratios. Changes in the observed price ratios are almost completely reflected in future returns. All variables are in logs, so a 1% increase

Table VI

Stock return predictability

This table shows the ability of real cash flow expectations to predict stock returns. The first and third columns show the coefficients from univariate linear regressions of ten-year nominal and real stock returns on the observed price-earnings ratio (pe_t) and price-dividend ratio (pd_t). The second and fourth columns show the coefficients from univariate linear regressions of ten-year nominal and real stock returns on the fundamental price-earnings ratio (pe_t^{fun}) and price-dividend ratio (pd_t^{fun}), which are constructed using real cash flow expectations and constant discount rates. For each regression, the table reports the slope coefficient β and the R^2. Small-sample adjusted Newey-West standard errors in parenthesis.

		Price-earnings ratio		Price-dividend ratio	
		Observed	Fundamental	Observed	Fundamental
Nominal returns $\sum_{j=1}^{10} r_{t+j}$	β	−0.78	−0.77	−0.88	−0.92
		(0.11)	(0.09)	(0.11)	(0.17)
	R^2	0.60	0.42	0.81	0.64
Real returns $\sum_{j=1}^{10} \tilde{r}_{t+j}$	β	−0.59	−0.54	−0.69	−0.69
		(0.14)	(0.13)	(0.15)	(0.19)
	R^2	0.45	0.26	0.65	0.46

in the price ratios is associated with a 0.78% and 0.88% decrease in future ten-year returns, respectively. Importantly, increases in the fundamental price ratios are also reflected in future returns, with nearly identical coefficients of −0.77 and −0.92, respectively. The second row shows the analogous results using real returns for the next ten years. Again, the coefficients for the observed price ratios, −0.59 and −0.69, are nearly identical to the coefficients for the fundamental price ratios, −0.54 and −0.69, and all coefficients are significant. In all cases, the differences between the coefficients for the observed price ratios and the fundamental price ratios are small and statistically insignificant.

Further, looking at the R^2 values, we see that the majority of the R^2 for the observed price ratios is generated by the fundamental price ratios. For example, the observed price-dividend ratio explains 81% of variation in future returns and the fundamental price-dividend ratio explains 64%. Thus, even if we attribute the entire difference to time-variation in discount rates, subjective expectations of real cash flows would still be the primary factor driving return predictability.

Combined, the results of Table V and Table VI imply that statistical forecasts of future returns based on observed price ratios are nearly identical to statistical forecasts based on fundamental price ratios. Fundamental price ratios move nearly 1-1 with observed price ratios with high R^2's and have virtually identical coefficients for predicting future returns. In other words, an econometrician using observed price ratios and an econometrician using subjective expectations of real cash flows produce almost the same forecasts for future returns. This means that rather than reflecting time-variation in investors' discount rates, movements in statistical forecasts of returns primarily reflect predictable errors in subjective expectations of inflation and nominal earnings growth.

C. Rejection of expectations hypothesis

The expectations hypothesis posits that the n-period yield $y_t^{(n)}$ is the expectation of the average one-period yield $y_{t+j}^{(1)}$ over the next n periods, plus a constant term premium.[21] Under RE, the expectations hypothesis implies that movements in yields must ultimately be related to movements in $y_{t+j}^{(1)}$ and the volatility of holding returns is constrained by the volatility of movements in the one-period yield. Specifically, equation (8) shows that under RE and the expectations hypothesis, the variance of holdings returns $h_{t+1}^{(n)}$ is bounded by $Var\left(y_t^{(1)}\right)$ scaled by a constant η. The first column of Table VII shows that this bound is decisively violated in the data by a factor of 5.6. Under RE, these large movements in $h_{t+1}^{(n)}$ soundly reject the expectations hypothesis.

In this section, we show that subjective expectations of inflation can quantitatively match this violation without using time-varying term premia. The fundamental yield $y_t^{(n),fun}$ is simply equal to the expected average inflation over the next n-periods plus a constant, meaning that the expectations hypothesis holds by definition. The holding return using fundamental yields is then measured from equation (3).[22] The second row of Table VII shows that fundamental yields violate this bound by a factor of 5.22, almost exactly matching the violation in the observed data. Using a one-sided F-test for equality of variance, we confidently reject that $Var\left(h_{t+1}^{(10)}\right)$ is below the bound for both the observed yields and the fundamental yields. We cannot reject that the two variance ratios are equal.

Equation (8) demonstrates how fundamental yields can violate this bound even though they satisfy the expectations hypothesis. Forecast errors that negatively comove with yields relax the bound, implying that the variance of holding returns under the expectations hypothesis can exceed $\eta Var\left(y_t^{(1)}\right)$. Intuitively, if long-term inflation expectations are too high when yields are high, then these inflation expectations will have to be revised downwards as investors realize their mistake. This downward revision in long-term inflation expectations lowers yields, which produces a high holding return following equation (3). To an econometrician assuming RE, it appears as if the high yield and the high holding return are due investors demanding a high term premium. Time-variation in these predictable inflation errors similarly makes it appear as if term premia are time-varying.

[21] Rejecting the expectations hypothesis is equivalent to stating that term premia $E_t^*\left[h_{t+1}^{(n)}\right] - y_t^{(1)}$ vary over time.

[22] Using the believed persistence ϕ_π^*, the holding return reduces to a constant plus $\frac{1-\phi_\pi^{*10}}{1-\phi_\pi^*}E_t^*\left[\pi_{t+1}\right] - \frac{1-\phi_\pi^{*9}}{1-\phi_\pi^*}E_{t+1}^*\left[\pi_{t+2}\right]$.

Table VII

Rejection of expectations hypothesis

This table compares the excess volatility of holding returns for the observed yields and the fundamental yields. The variance ratio measures the variance of one-period holding returns on a ten-year bond divided by the variance of the one-period yield multiplied by a constant $\eta = 10^2/(20-1)$. Under RE and the expectations hypothesis, this ratio has an upper bound of 1. The first column shows this ratio in the observed detrended data. The second column shows the variance ratio based on fundamental yields constructed using detrended inflation expectations and constant discount rates. The second row shows the significance of the one-sided F-test for equality of variance adjusted for autocorrelation of samples as in Priestley (1981).

	Observed yields	Fundamental yields
$Var\left(h_{t+1}^{(10)}\right)/\eta Var\left(y_t^{(1)}\right)$	5.60	5.22
F-test	0.00001	0.00046

The results of Table VII indicate that the excess volatility of holding returns can be explained as a rejection of RE rather than a rejection of the expectations hypothesis. This is consistent with the results of Piazzesi, Salomao, and Schneider (2015), who find that survey expectations of the term premia $E_t^*\left[h_{t+1}^{(n)}\right] - y_t^{(1)}$ are relatively flat compared statistical measures of term premia. This is also consistent with Barr and Campbell (1997), who find that holding returns on inflation protected bonds are much less volatile than holding returns on nominal bonds.

IV. Fundamental Extrapolation Model

In this section, we show that a model of fundamental extrapolation closely matches the results of our diagnostic test. Importantly, the impact of fundamental extrapolation depends on the objective persistence of the variable being forecasted. Because of this, the model successfully reproduces the stark differences in the term structure of subjective expectations and errors for inflation and nominal earnings growth documented in Section II. For a persistent variable, such as inflation, errors are concentrated in long-term expectations. For a transitory variable, such as nominal earnings growth, errors are concentrated in short-term expectations. Despite not using any price information in the estimation, the model is also quantitatively successful in matching the comovements of expectations and realizations with bond yields and the price-earnings ratio. As further support for the fundamental extrapolation model, we find that it accurately matches the evolution of survey expectations over time. Given the time series of realized inflation and nominal earnings growth for 1976-2018, the model closely replicates the time series of both short-term and long-term expectations for inflation and nominal earnings growth.

A. Model

Given that we are only interested in variances and covariances, we demean all variables without loss of generality. Inflation and nominal earnings growth are objectively AR(1) processes,

$$\pi_t = \phi_\pi \pi_{t-1} + \upsilon_{t,\pi} \tag{18}$$

$$\Delta e_t = \phi_e \Delta e_{t-1} + \upsilon_{t,e} \tag{19}$$

with variances σ_π^2, σ_e^2 and covariance $\sigma_{\pi,e}$.

Agents are extrapolative and form their beliefs based on a weighted sum of current and past realizations,

$$\omega_{t,\pi} = \sum_{j=0}^{\infty} \beta_\pi^j \pi_{t-j},$$

where β_π determines the relative weight placed on older and more recent observations. Specifically, agents believe that inflation follows

$$\pi_{t+1} = (\phi_\pi^* - \beta_\pi)\, \omega_{t,\pi} + \upsilon_{t,\pi}^*. \tag{20}$$

From equation (20), agents' short-term and long-term expectations are

$$E_t^*[\pi_{t+1}] = \phi_\pi^* \pi_t + \beta_\pi \left(E_{t-1}^*[\pi_t] - \pi_t\right) \tag{21}$$

$$E_t^*[\pi_{t+1+j}] = \phi_\pi^{*j} E_t^*[\pi_{t+1}]. \tag{22}$$

In other words, the parameter ϕ_π^* is the believed persistence as defined in Section II.

We use an analogous framework for nominal earnings growth expectations. Agents are extrapolative with parameters β_e and ϕ_e^* and have expectations

$$E_t^*[\Delta e_{t+1}] = \phi_e^* \Delta e_t + \beta_e \left(E_{t-1}^*[\Delta e_t] - \Delta e_t\right) \tag{23}$$

$$E_t^*[\Delta e_{t+1+j}] = \phi_e^{*j} E_t^*[\Delta e_{t+1}]. \tag{24}$$

Last, agents have constant discount rates, which means that bond yields and the stock price-earnings ratio are simply

$$y_t^{(n)} = \frac{1}{n}\frac{1-\phi_\pi^{*n}}{1-\phi_\pi^*} E_t^*[\pi_{t+1}] \tag{25}$$

$$pe_t = \frac{1}{1-\rho\phi_e^*} E_t^*[\Delta e_{t+1}] - \frac{1}{1-\rho\phi_\pi^*} E_t^*[\pi_{t+1}]. \tag{26}$$

B. Diagnostic test

Notably, without using any information on observed prices, this model successfully replicates the comovement of bond yields and the price-earnings ratio with both expected and realized

Table VIII

Estimation of model parameters

This table shows the estimated model parameters. The objective parameters $\phi_\pi, \phi_e, \sigma_\pi, \sigma_e$ and $\sigma_{\pi,e}$ are estimated from realized inflation π_t and nominal earnings growth Δe_t over our 1976-2018 sample. The subjective parameters ϕ^*_π, ϕ^*_e are estimated from equations (22) and (24) using both short-term and long-term survey expectations. After estimating ϕ^*_π, ϕ^*_e, we calculate $\beta_\pi$and β_e by regressing $E^*_t[\pi_{t+1}] - \phi^*_\pi \pi_t$ on the surprise $E^*_{t-1}[\pi_t] - \pi_t$ and regressing $E^*_t[\Delta e_{t+1}] - \phi^*_e \Delta e_t$ on the surprise $E^*_{t-1}[\Delta e_t] - \Delta e_t$. Small-sample adjusted Newey-West standard errors in parenthesis.

ϕ_π	ϕ_e	σ_π	σ_e	$\sigma_{\pi,e}$	ϕ^*_π	ϕ^*_e	β_π	β_e
0.76	-0.20	0.027	0.38	0.001	0.96	0.004	0.69	0.74
(0.14)	(0.14)	(0.002)	(0.05)	(0.001)	(0.01)	(0.075)	(0.03)	(0.02)

inflation and nominal earnings growth at both short and long horizons. Importantly, the model not only matches the qualitative features of these comovements, but quantitatively replicates their magnitudes.

All model parameters are estimated from real cash flow data. The objective persistences ϕ_π, ϕ_e, variances σ^2_π, σ^2_e, and covariance $\sigma_{\pi,e}$ are estimated from realized inflation and nominal earnings growth. The believed persistences ϕ^*_π, ϕ^*_e are estimated from equations (22) and (24) using short-term and long-term survey expectations, just as in Section II. Finally, the weights β_π and β_e are estimated from equations (21) and (23) using short-term survey expectations and realized forecast errors $\pi_t - E^*_{t-1}[\pi_t]$, $\Delta e_t - E^*_{t-1}[\Delta e_t]$. Table VIII shows the parameter values.

Figure 5 shows the comovement of prices with expected and realized inflation and nominal earnings growth. The dark bars show the values measured in the data and the light bars show the values in the model. Along with showing that the model comovements match the qualitative patterns observed in the data, Figure 5 shows the dark and light bars side-by-side to emphasize that the model also quantitatively matches the size of these comovements. These comovements represent twelve untargeted moments and, in every case, we cannot reject that the value measured in the data exactly equals the model prediction.

The top Panel of Figure 5 shows the comovement of the price-earnings ratio with inflation. In the model, expected and realized short-term inflation have small, negative comovements with the price-earnings ratio that are virtually identical. For expected and realized long-term inflation, these comovements are scaled by $(1 - \phi^{*10}_\pi) / (1 - \phi^*_\pi)$ and $(1 - \phi^{10}_\pi) / (1 - \phi_\pi)$, respectively. Because the believed persistence is higher than the objective persistence, expected long-term inflation comoves much more with the price-earnings ratio than realized long-term inflation, matching the large error that we observe in the data. The bottom panel shows the same pattern for the comovement of the ten-year yield with inflation. Importantly, despite not using any information about the variance of yields relative to the variance of the price-earnings ratio in the estimation, the model closely replicates the magnitudes of the

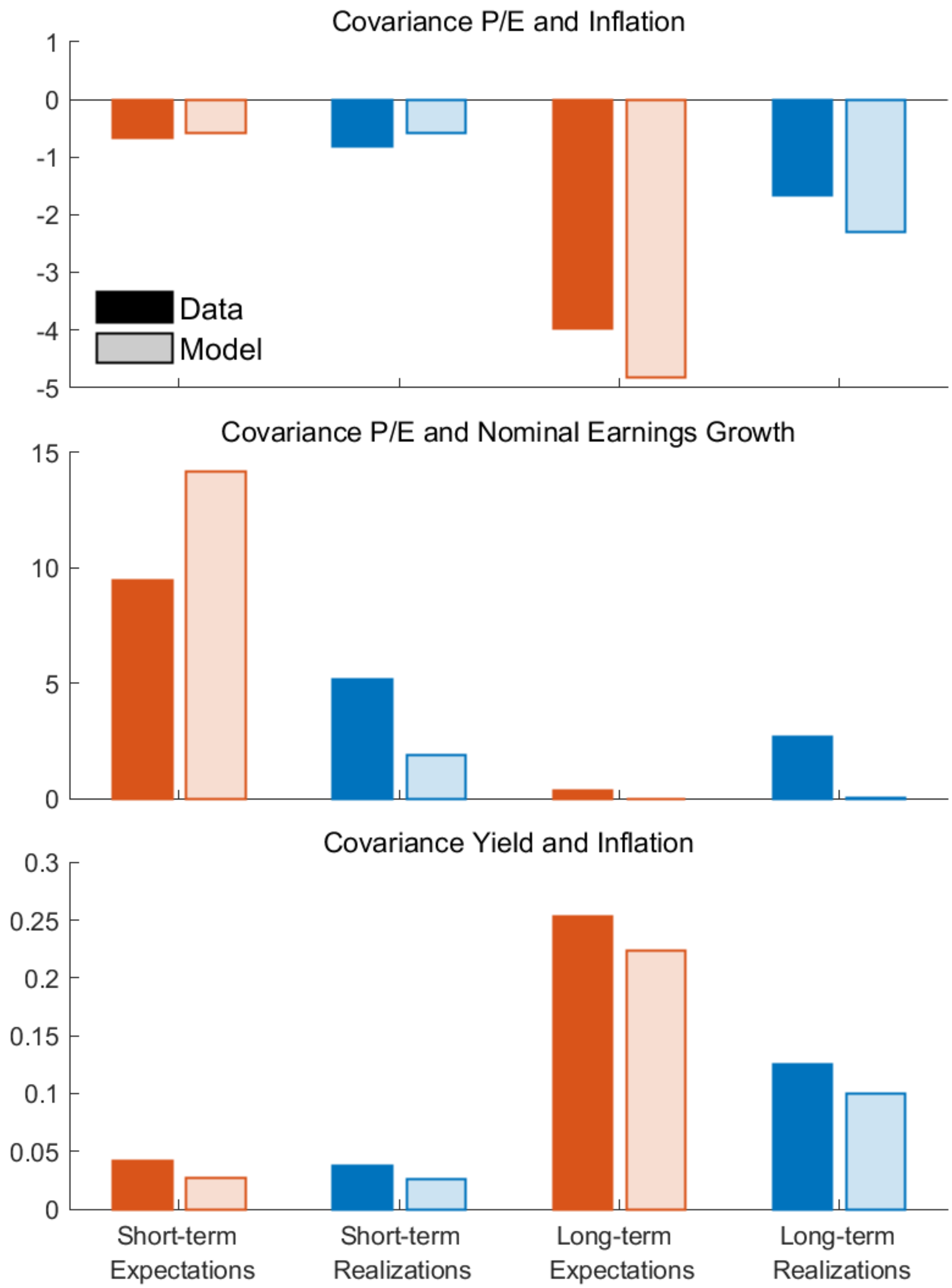

Figure 5. Matching diagnostic test. This figure shows the covariance of prices with expected and realized real cash flows in the data and in the estimated model. The empirical covariances are shown in dark-colored bars and the model covariances are shown in light-colored bars. The top panel shows the covariances of the S&P 500 price-earnings ratio with expected and realized inflation at different horizons. The middle panel shows the covariances of the S&P 500 price-earnings ratio with expected and realized nominal earnings growth at different horizons. The bottom panel shows the covariances of the 10-year Treasury yield with expected and realized inflation at different horizons.

comovements with both prices.

The middle panel of Figure 5 shows the comovement of the price-earnings ratio with expected and realized nominal earnings growth. Consistent with the data, the model produces a large comovement between short-term expectations and the price-earnings ratio and almost no comovement between long-term expectations and the price-earnings ratio. This is driven by the low believed persistence ϕ_e^*. While the model understates the comovement of realized short-term and long-term nominal earnings growth with the price-earnings ratio, these comovements are both statistically insignificant in data. As a result, the model still accurately captures the fact that in the data there is no significant comovement between the price-earnings ratio and forecast errors for long-term nominal earnings growth. Instead, in both the model and the data, errors in expectations are concentrated in short-term expectations which comove much more with the price-earnings ratio than realized short-term nominal earnings growth.

The model also matches the results of Section II.C and Table II which emphasize the importance of the high believed persistence in long-term inflation expectations and the response of short-term nominal earnings growth expectations to surprises. As shown in equation (22), movements in model long-term inflation expectations are driven by changes in $\frac{1-\phi_\pi^{*10}}{1-\phi_\pi^*}E_t^*\left[\pi_{t+1}\right]$ where $\frac{1-\phi_\pi^{*10}}{1-\phi_\pi^*}$ is more than double the objective value $\frac{1-\phi_\pi^{10}}{1-\phi_\pi}$.[23] Similarly, given that ϕ_e^* is near 0, equation (23) shows that movements in model short-term nominal earnings growth expectations are due to the response to surprises $\beta_e\left(E_{t-1}^*\left[\Delta e_t\right]-\Delta e_t\right)$. Thus, fundamental extrapolation gives an intuitive framework that captures both of these forces. For comparison, sticky information or diagnostic expectations would allow subjective expectations to depend on surprises but do not explain the high believed persistence for inflation as they imply that the believed persistence equals the objective persistence, i.e. $E_t^*\left[\pi_{t+1+j}\right]=\phi_\pi^j E_t^*\left[\pi_{t+1}\right]$. This is shown formally in Appendix D.1. Similarly, a model where agents make mistakes about persistences such as Alti and Tetlock (2014) would allow for a high ϕ_π^* but would not allow short-term nominal earnings growth expectations to depend on surprises.

While a model of sticky information would not match the high believed persistence ϕ_π^*, our model of fundamental extrapolation is still consistent with the findings on sticky information of Coibion and Gorodnichenko (2015). Looking at equation (21), the parameter β_π causes agents' expectations to depend less on current inflation π_t and more on their past

[23] The differences between the objective and believed persistence are particularly pronounced at longer horizons. When estimated at a one-year horizon, the persistence of inflation is 0.76 (0.14) which could overlap with the believed persistence ϕ_π^* of 0.96. However, when estimated using average inflation over the next ten years, the annual persistence of inflation is 0.76 (0.05) which is significantly lower than the believed persistence. The estimated value for the persistence is consistent across horizons but the standard error of the estimate is much smaller when longer horizons are used.

expectations $E_{t-1}^{*}\left[\pi_t\right]$. As a result, the parameter β_π maps almost exactly to the sticky information coefficient of Coibion and Gorodnichenko (2015). Using the formal test for sticky information, a regression of forecast errors $\pi_{t+1} - E_t^{*}\left[\pi_{t+1}\right]$ on revisions $E_t^{*}\left[\pi_{t+1}\right] - E_{t-1}^{*}\left[\pi_{t+1}\right]$ in the model implies a sticky information coefficient of 0.65.[24] Further, we cannot reject that our model value of 0.65 matches the sticky information coefficient of 0.54 (0.10) measured in Coibion and Gorodnichenko (2015). As shown in the Appendix D.2, the fundamental extrapolation model nests sticky information, meaning that there is no tension between our findings and previous evidence of stickiness. While we find that errors in model short-term inflation expectations do not contribute to price variation, these expectations are still consistent with important previous evidence on stickiness.

C. Intuition for term structure of errors

A key result of the model is that it replicates our findings on the term structure of errors in subjective expectations. Namely, current prices mainly comove with errors in long-term rather than short-term inflation expectations and comove with errors in short-term rather than long-term nominal earnings growth expectations.

To avoid repetition, let x be a general variable that represents inflation or nominal earnings growth. In the model, rational expectations are

$$E_t\left[x_{t+j}\right] = \phi_x^j x_t \tag{27}$$

Agents' subjective expectations are rational when $\phi_x^* - \phi_x$ and β_x are both zero. As shown in Table VIII, the parameters for subjective expectation biases are almost identical for inflation and nominal earnings growth. For both inflation and nominal earnings growth, $\phi_x^* - \phi_x$ is 0.20 and β_x is very similar at 0.69 for inflation and 0.72 for nominal earnings growth. As a result, the stark difference in the term structure of errors for inflation and nominal earnings growth expectations is not due to a difference in biases. Instead, both inflation and nominal earnings growth expectations demonstrate virtually identical biases and the difference in the term structure of errors is driven by the difference in the objective persistence ϕ_x. For a persistent process like inflation, these biases generate errors that are concentrated in long-term expectations and for a transitory process like nominal earnings growth, these biases generate errors that are concentrated in short-term expectations.

For intuition, we consider each effect in isolation. When $\phi_x^* - \phi_x$ is zero, errors in subjective expectations are

$$E_t^{*}\left[x_{t+j}\right] - E_t\left[x_{t+j}\right] = \phi_x^{j-1}\beta_x\left(E_{t-1}^{*}\left[x_t\right] - x_t\right). \tag{28}$$

[24] The regression coefficient is $\frac{\lambda}{1-\lambda}$ where λ is the sticky information coefficient.

If the objective persistence ϕ_x is small, then the error is primarily concentrated in short-term expectations as the ϕ_x^{j-1} coefficient quickly goes to zero as the horizon increases. Conversely, if the objective persistence is large, then the error is mainly concentrated in longer horizon expectations, as errors in short-term expectations comprise only a small portion of the total $\frac{1}{1-\phi_x}\beta_x\left(E_{t-1}^*\left[x_t\right]-x_t\right)$ error across all horizons. The second isolated effect occurs when β_x is zero, in which case errors in subjective expectations are

$$E_t^*\left[x_{t+j}\right]-E_t\left[x_{t+j}\right] \;=\; \left(\phi_x^{*j}-\phi_x^j\right)x_t. \tag{29}$$

Once again, if the process is objectively transitory, then a small bias in $\phi_x^*-\phi_x$ primarily generates errors in short-term expectations as the $\phi_x^{*j}-\phi_x^j$ coefficient quickly goes to zero as the horizon increases. If the process is objectively persistent, then the error is mainly concentrated in longer horizon expectations. For example, given our estimated values of 0.96 and 0.76, $\phi_\pi^*-\phi_\pi$ is 0.2 and $\phi_\pi^{*10}-\phi_\pi^{10}$ is 0.6. In short, the difference between a persistent process and a very persistent process only becomes clear at long horizons.

More generally, when $\phi_x^*-\phi_x$ and β_x are both potentially non-zero, errors in subjective expectations are

$$E_t^*\left[x_{t+1+j}\right]-E_t\left[x_{t+1+j}\right] \;=\; \phi_x^j\left(E_t^*\left[x_{t+1}\right]-E_t\left[x_{t+1}\right]\right)+\left(\phi_x^{*j}-\phi_x^j\right)E_t^*\left[x_{t+1}\right]. \tag{30}$$

Again, this directly shows that if ϕ_x and ϕ_x^* are small, then errors in expectations will quickly disappear as the horizon increases. Conversely, for a high ϕ_x and ϕ_x^*, long-term expectations can have large errors even if there are no errors in one-year expectations. Even if one-year subjective expectations $E_t^*\left[x_{t+1}\right]$ are close or even equal to rational expectations $E_t\left[x_{t+1}\right]$, errors in the believed persistence will still cause longer horizon subjective expectations $\phi_x^{*j-1}E_t^*\left[x_{t+1}\right]$ to deviate noticeably from the rational $\phi_x^{j-1}E_t\left[x_{t+1}\right]$.

D. *Replicating survey expectations*

In addition to matching the results of the diagnostic test, we also find that fundamental extrapolation accurately replicates the dynamics of survey expectations over time. Specifically, equations (21)-(24) closely match the observed evolution in both short-term and long-term survey expectations for inflation and nominal earnings growth over the last 40 years. Importantly, these equations not only qualitatively match movements in survey expectations but also quantitatively match the magnitude of these changes both across time and across forecast horizons.

We first consider survey and model inflation expectations. Using just two parameters (ϕ_π^*,β_π), we can closely replicate survey inflation expectations solely from the realized inflation series. We set the initial model surprise $E_{t-1}^*\left[\pi_t\right]-\pi_t$ to match survey expectations in

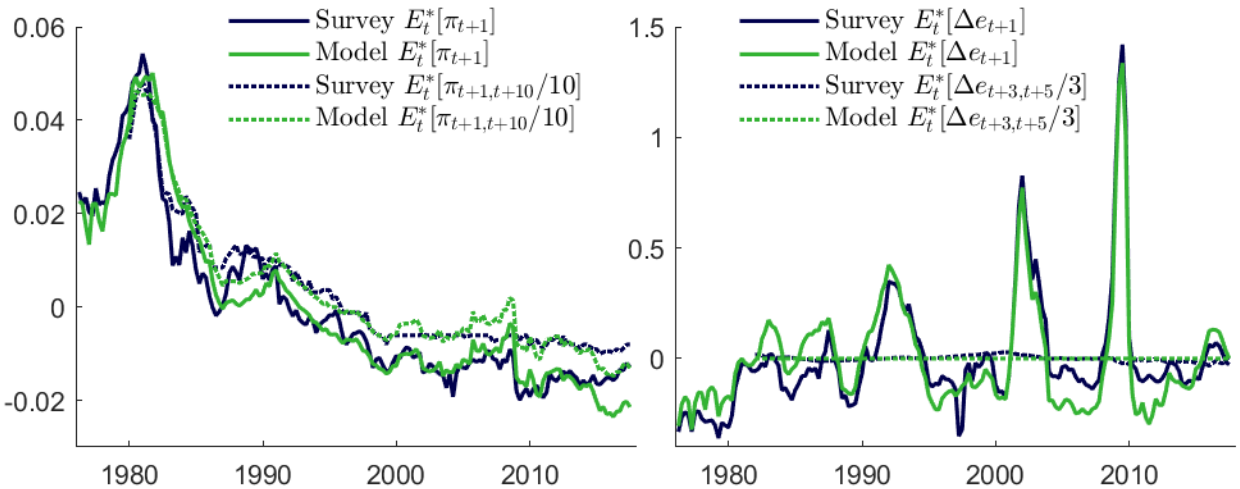

Figure 6. Replicating survey expectations. The left panel compares the survey expectations (blue) for inflation and to the model expectations (green) calculated from realized inflation and ϕ_π^*, β_π. Short-term expectations are shown in solid lines and long-term expectations are shown in dashed lines. The right panel shows the same comparison for survey nominal earnings growth expectations and the model expectations calculated from realized nominal earnings growth and ϕ_e^*, β_e. We demean all series without loss of generality.

1976 and then construct the model one-year inflation expectations using realized inflation π_t and equation (21).[25] Model expectations for average ten-year inflation are then given by equation (22). The left panel of Figure 6 compares survey expectations for short-term and long-term inflation to the model expectations. The model accurately replicates the large changes in both short-term and long-term inflation expectations, including the spike from the late 1970's to the early 1980's, the gradual decline from the 1980's to the late 1990's, and the leveling off of expectations after 2000.

Similarly, the two parameters (ϕ_e^*,β_e) provide a parsimonious description of survey nominal earnings growth expectations. Repeating the procedure used for inflation expectations, we replicate the survey expectations solely using realized nominal earnings growth. The initial model surprise $E_{t-1}^*[\Delta e_t] - \Delta e_t$ is set to match survey expectations in 1976 and model one-year expectations are constructed using equation (23) and realized nominal earnings growth. Model long-term expectations are constructed from equation (24). The right panel of Figure 6 shows that the model short-term expectations match almost all variation in survey short-term expectations, including the massive spikes in expected earnings growth during the dot-com bust and the financial crisis. Additionally, the model expectations accurately capture that most movements in nominal earnings growth expectations occur in short-term expectations, with long-term expectations being relatively flat compared to short-term ex-

[25]We focus on the recursive equation (21) since the infinite sum in equation (20) is not observable.

pectations.

V. Conclusion

In this paper, we demonstrate when deviations from standard rational expectations (RE) help to explain price variation and return puzzles. Using accounting identities, we show that the necessary and sufficient condition is that forecast errors for subjective expectations must comove with current prices. This provides a simple and easily implementable diagnostic test to determine where the relevant deviations from RE do and do not occur in subjective expectations.

Applying this diagnostic test to survey expectations, we find that systematic errors in expectations of long-term inflation and short-term nominal earnings growth are the main driver of price variation and return puzzles for aggregate bond and stock markets. In contrast, expectations of short-term inflation and long-term nominal earnings growth do not appear to play a role in either market despite containing systematic deviations from RE. These results are all consistent with a model of fundamental extrapolation where agents price assets using constant discount rates and biased expectations of future real cash flows. In this model, fundamental extrapolation leads to errors primarily in long-term expectations for an objectively persistent process such as inflation and leads to errors primarily in short-term expectations for an objectively transitory process such as nominal earnings growth.

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Appendix

A. Derivation of formulas

In order to arrive to equation (1) we start with the one-year return identity

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{\left(\frac{P_{t+1}}{D_{t+1}} + 1\right)\frac{D_{t+1}}{D_t}}{\frac{P_t}{D_t}},$$

where P_t and D_t represent the current nominal price and nominal dividends of the S&P 500 index. Log-linearizing around a long-term average of P/D, we can state the price-dividend ratio pd_t in terms of future dividend growth, Δd_{t+1}, future returns, r_{t+1}, and the future price-dividend ratio, pd_{t+1}, all in logs:

$$r_{t+1} \quad = \quad \kappa^d + \Delta d_{t+1} - pd_t + \rho pd_{t+1}, \tag{A1}$$

where κ^d is a constant, $\rho = e^{\bar{pd}} / \left(1 + e^{\bar{pd}}\right) < 1$ and $\bar{pd}$ is the mean value of the log price-dividend ratio. Using the log payout ratio de_t, we can insert the identity $pe_t = pd_t + de_t$[26] into (A1) to obtain

$$\tilde{r}_{t+1} \quad \approx \quad \kappa + \Delta\tilde{e}_{t+1} - pe_t + \rho pe_{t+1} \tag{A2}$$

where we approximate $(1-\rho)\, de_{t+1}$ as 0 given that $1-\rho$ is close to 0.

To establish identities (5) and (6) we just need to use the fact that forecast errors are unpredictable with information at time t under RE, meaning that

$$Cov(pe_t, \Delta\tilde{e}_{t+j}) \quad = \quad Cov\left(pe_t, E_t\left[\Delta\tilde{e}_{t+j}\right]\right). \tag{A3}$$

Under subjective expectations, the predictability of forecast errors with the price-earnings ratio is:

$$Cov(pe_t, f^{\Delta\tilde{e}_{t+j}}) \quad = \quad Cov(pe_t, \Delta\tilde{e}_{t+j}) - Cov\left(pe_t, E_t^*\left[\Delta\tilde{e}_{t+j}\right]\right) \tag{A4}$$

Equation (A3) and (A4) lead to equation (5) which expresses the comovement of price-earnings ratio with subjective real earnings growth expectations in terms of its comovement with expectations under RE and the predictability of forecast errors for real earnings growth. In a similar fashion, the covariance of the 10-year bond yield with subjective inflation expectations can be expressed in terms of its covariance with expectations under RE and the predictability of forecast errors for inflation as in equation (6).

To establish identity (7), we start from the fact that equation (2) is satisfied with and

[26]Because we are using the aggregate S&P 500, we do not need to worry about very small or negative values for earnings.

without applying expectations. Hence

$$\sum_{j=1}^{\infty}\rho^{j-1}\tilde{r}_{t+j}-\sum_{j=1}^{\infty}\rho^{j-1}E_t^*\left[\tilde{r}_{t+j}\right] \;=\; \sum_{j=1}^{\infty}\rho^{j-1}\Delta\tilde{e}_{t+j}-\sum_{j=1}^{\infty}\rho^{j-1}E_t^*\left[\Delta\tilde{e}_{t+j}\right].$$

By covarying both sides with the price-earnings ratio we arrive to equation (7). Finally, to establish the inequality in equation (8) we generalize the Shiller (1979) variance bound. From equation (3) we have that the variance of one-period holding returns can be expressed as

$$Var\left(h_{t+1}^{(n)}\right) \;=\; n^2Var\left(y_t^{(n)}\right)+(n-1)^2\,Var\left(y_{t+1}^{(n-1)}\right)-2n\,(n-1)\,Cov\left(y_t^{(n)},y_{t+1}^{(n-1)}\right).$$

The covariance term in the above equation can be expressed as

$$(n-1)\,Cov\left(y_{t+1}^{(n-1)},y_t^{(n)}\right) \;=\; nVar\left(y_t^{(n)}\right)-Cov\left(y_t^{(1)},y_t^{(n)}\right)-Cov\left(h_{t+1}^{(n)}-y_t^{(1)},y_t^{(n)}\right)$$

and $Cov\left(h_{t+1}^{(n)}-y_t^{(1)},y_t^{(n)}\right)$ can be expressed as

$$\begin{aligned}Cov\left(h_{t+1}^{(n)}-y_t^{(1)},y_t^{(n)}\right) &= Cov\left(E_t^*\left[h_{t+1}^{(n)}\right]-y_t^{(1)},y_t^{(n)}\right)+Cov\left(f_t^{h_{t+1}^{(n)}},y_t^{(n)}\right)\\ &= Cov\left(E_t^*\left[h_{t+1}^{(n)}\right]-y_t^{(1)},y_t^{(n)}\right)-Cov\left(\sum_{j=2}^{n}f_t^{h_{t+j}^{(n+1-j)}},y_t^{(n)}\right).\end{aligned}$$

We plug these two values back into the variance of $h_{t+1}^{(n)}$ to get

$$\begin{aligned}Var\left(h_{t+1}^{(n)}\right) &= -n^2Var\left(y_t^{(n)}\right)+(n-1)^2\,Var\left(y_{t+1}^{(n-1)}\right)+2nCov\left(y_t^{(1)},y_t^{(n)}\right)\\ &\quad +2nCov\left(E_t^*\left[h_{t+1}^{(n)}\right]-y_t^{(1)},y_t^{(n)}\right)-2nCov\left(\sum_{j=2}^{n}f_t^{h_{t+j}^{(n+1-j)}},y_t^{(n)}\right).\end{aligned} \quad \text{(A5)}$$

Under RE and the expectations hypothesis, the fourth and fifth term of the equation would be zero. The hypothetical variance under these conditions, denoted as $\overline{Var}\left(h_{t+1}^{(n)}\right)$ is

$$\overline{Var}\left(h_{t+1}^{(n)}\right) \;=\; -n^2Var\left(y_t^{(n)}\right)+(n-1)^2\,Var\left(y_{t+1}^{(n-1)}\right)+2nCov\left(y_t^{(1)},y_t^{(n)}\right).$$

From Shiller (1979), we know that under these conditions the variance has an upper bound defined by

$$\overline{Var}\left(r_{t+1}^{(n)}\right) \;\leq\; \frac{n^2}{2n-1}Var\left(y_t^{(1)}\right).$$

This means that the generalized bound from (A5) is

$$Var\left(h_{t+1}^{(n)}\right) \;\leq\; \frac{n^2}{2n-1}Var\left(y_t^{(1)}\right)+2nCov\left(E_t^*\left[h_{t+1}^{(n)}\right]-y_t^{(1)},y_t^{(n)}\right)-2nCov\left(\sum_{j=2}^{n}f_t^{h_{t+j}^{(n+1-j)}},y_t^{(n)}\right)$$

which reduces to equation (8) under the expectations hypothesis as expected term premia are constant.

B. *Data sources for Inflation and Nominal Earnings Growth Expectations*

B.1. Inflation Expectations

Our main source for inflation expectations is the Survey of Professional Forecasters (SPF). The SPF contains quarterly median inflation forecasts of one-year inflation expectations from 1976Q1-2018Q2 and average ten-year inflation expectations from 1991Q1-2018Q2. As suggested by the technical documentation in the SPF, the average ten-year inflation expectation series is complemented backwards to 1979Q3 with the average ten-year inflation expectations from the Philadelphia Fed's Livingston Survey and from the Blue Chip Economic Indicators Survey.

For robustness, we also analyze an alternative measure of inflation expectations from the Michigan Survey of Consumers which has both one-year inflation expectations and five-to-ten-year average inflation expectations. We utilize the same 1976Q1-2018Q2 sample that is studied for the SPF expectations and find very similar results both qualitatively and quantitatively.

B.2. Nominal Earnings Growth Expectations

We construct one-year cash flow expectations for the S&P 500 index following the aggregating procedure in De la O and Myers (2021). The Summary Statistics of the Thomson Reuters I/B/E/S database contains the median analyst forecasts for EPS (earnings per share) since 1976.[27] Using these individual forecasts, we measure aggregate earnings expectations using the constituents of the S&P 500 at each point in time. Expected earnings growth for the S&P 500 is then simply measured from the expectation of one-year aggregate earnings for the S&P 500 and the current earnings of the S&P 500. The online appendix of De la O and Myers (2021) shows in detail the tests of using this methodology for the I/B/E/S database.

For longer horizons, analysts forecast growth rather than the future level of earnings. For any set of firms, the growth of total earnings for the entire set is equal to the earnings-weighted average of individual firm earnings growth. Thus, following Nagel and Xu (2021), we measure the expected long-term earnings growth of the S&P 500 using the earnings-weighted average of the individual firm forecasts, dropping firms with negative weights. To ensure few firms are dropped, we weight by 5-year average earnings as these are rarely negative. However, we do not find any noticeable change in our results if we weight by forecasted earnings for the end of the fiscal year as in Nagel and Xu (2021).

[27] Using the mean forecasts does not change the results in any noticeable way.

Table AI

Diagnostic test for Michigan inflation expectations

This table shows the results of the diagnostic test which determines if differences between subjective expectations and RE explain asset price movements and return puzzles. The condition is that prices must comove with forecast errors. The first row shows the covariance of the S&P 500 price-earnings ratio with short-term inflation expectations $E_t^*[\pi_{t+1}]$, realized short-term inflation π_{t+1}, and the forecast errors $\pi_{t+1} - E_t^*[\pi_{t+1}]$ from 1978Q1 to 2018Q2. The second row shows the covariance of the S&P 500 price-earnings ratio with long-term inflation expectations $E_t^*[\pi_{t+1,t+10}]$, realized long-term inflation $\pi_{t+1,t+10}$, and the forecast errors $\pi_{t+1,t+10} - E_t^*[\pi_{t+1,t+10}]$ using quarterly data from 1979Q1 to 2018Q2. The third and fourth rows show analogous results using the ten-year Treasury yield instead of the S&P 500 price-earnings ratio. The fifth and sixth rows show analogous results using the S&P 500 price-dividend ratio. Expectations expressed in percentages. We use small-sample adjusted Newey-West standard errors. Superscripts indicate significance at the 1% (***), 5% (**), and 10% (*) level.

	Horizon	Expected	Realized	Forecast Error
$Cov(pe_t, \cdot)$	Short-term (π_{t+1})	-0.61^{***}	-0.74^{**}	-0.13
	Long-term ($\pi_{t+1,t+10}$)	-4.45^{**}	-1.83^{**}	2.62^{***}
$Cov\left(y_t^{(10)}, \cdot\right)$	Short-term (π_{t+1})	0.03^{***}	0.04^{***}	0.01
	Long-term ($\pi_{t+1,t+10}$)	0.28^{***}	0.13^{***}	-0.14^{***}
$Cov(pd_t, \cdot)$	Short-term (π_{t+1})	-0.56^{***}	-0.70^{**}	-0.15
	Long-term ($\pi_{t+1,t+10}$)	-5.82^{***}	-2.46^{***}	3.36^{***}

C. *Robustness checks*

C.1. Diagnostic test with Michigan inflation expectations

Our findings are virtually identical using short-term and long-term inflation expectations from the Michigan survey rather than the Survey of Professional Forecasters. As shown in Figure 1, short-term and long-term expectations from the SPF have similar volatilities with standard deviations of 1.8% and 1.2%, respectively. The results are virtually identical for the Michigan survey, where short-term and long-term expectations have standard deviations of 1.8% and 1.4%, respectively. Further, from equation (11), we estimate a believed persistence of ϕ_π^* of 0.95 (0.02) which almost exactly matches our estimate of 0.96 (0.01) from the SPF expectations.

Most importantly, we find the same results for our diagnostic test using the Michigan expectations. Table AI shows the results. The SPF results are shown in Tables I and IV.

C.2. Overlapping observations and Bauer and Hamilton (2018)

Overlapping forecast horizons can increase the persistence of forecast errors for long-term inflation. Because of this, we use Newey-West standard errors to calculate significance in Table I to account for any autocorrelation. For additional robustness, in this section we also directly calculate the significance under the worst-case scenario for overlapping observations. We do this following the methodology proposed by Bauer and Hamilton (2018). The general concept is that we run simulations to measure how often we spuriously find a comovement between forecast errors and current prices as large as what we observe in the data. Below, we describe the process for the price-earnings ratio. We then repeat the same process for the ten-year yield as well as the price-dividend ratio.

The price-earnings ratio is simulated as an AR(1) process, where the persistence is set equal to the observed persistence over our sample. The variance of shocks to the price-earnings ratio is set to match the observed variance of the price-earnings ratio. Additionally, the initial value of the simulated price-earnings ratio is set equal to the initial value observed in our data to account for any drift back to the mean which may generate trends in the price-earnings ratio over the sample. For example, if the price-earnings ratio is substantially below its mean at the beginning of the sample, then reversion to the mean will create an upward trend in the price-earnings ratio over time. We then simulate one-period forecast errors under the null hypothesis that forecast errors are unpredictable, i.e. $\pi_{t+j} - E_t^* [\pi_{t+j}]$ is white noise. Forecast errors for long-term inflation are then

$$\begin{aligned} f_{t+n} &\equiv \pi_{t+1,t+n} - E_t^* [\pi_{t+1,t+n}] \\ &= \sum_{j=1}^{n} \left(\pi_{t+j} - E_t^* [\pi_{t+j}]\right). \end{aligned}$$

If subjective expectations change over time, then there will be little overlap in the long-term forecast errors. For example, if $E_t^* [\pi_{t+2}]$ is very different from $E_{t+1}^* [\pi_{t+2}]$, then there is little similarity between the second term of f_{t+n}, which is $\pi_{t+2} - E_t^* [\pi_{t+2}]$, and the first term of f_{t+1+n}, which is $\pi_{t+2} - E_{t+1}^* [\pi_{t+2}]$. However, in the worst-case scenario where expectations do not change, then there is perfect overlap. Formally, when $\pi_{t+j} - E_t^* [\pi_{t+j}] = \pi_{t+j} - E_{t+k}^* [\pi_{t+j}]$ for all $k < j$, then f_{t+n} is an MA$(n-1)$ process. This causes f_{t+n} to be persistent which increases the probability of spuriously finding a large comovement between the price-earnings ratio and f_{t+n}.

For our analysis, we consider the worst-case scenario, where there is perfect overlap and where each period is only a quarter rather than a year. This second assumption means that f_{t+n} is MA(39) rather than MA(9) for ten-year inflation, which substantially raises the persistence. We set the variance of forecast errors to match the variance observed in the data. We run 100,000 simulations to estimate the probability of spuriously observing a covariance between the price-earnings ratio and long-term forecast errors as large as what we observe in the data.

For the price-earnings ratio, we estimate a p-value of 0.015. Repeating the process for the ten-year yield and price-dividend ratio gives p-values of 0.014 and 0.005, respectively. In short, even under the worst-case scenario, the likelihood of spuriously finding comovements as large as what we observe in the data is quite low.

C.3. Generalized persistence structure

We calculate a more generalized version of equations (11) and (12) that allows inflation expectations to impact nominal earnings growth expectations,

$$\begin{pmatrix} E_t^*\left[\pi_{t+1+j}\right] \\ E_t^*\left[\Delta e_{t+1+j}\right] \end{pmatrix} = \begin{pmatrix} \alpha_{\pi,j} \\ \alpha_{e,j} \end{pmatrix} + \begin{pmatrix} \phi_\pi & 0 \\ \phi_{\pi,e} & \phi_e \end{pmatrix}^j \begin{pmatrix} E_t^*\left[\pi_{t+1}\right] \\ E_t^*\left[\Delta e_{t+1}\right] \end{pmatrix} + \begin{pmatrix} \varepsilon_{t,j}^{\pi} \\ \varepsilon_{t,j}^{e} \end{pmatrix}. \quad \text{(A1)}$$

This generalization has no impact on our estimate of ϕ_π of 0.96 (0.01). Using the short-term and long-term inflation expectations and nominal earnings growth expectations, we estimate ϕ_e as 0.001 (0.088) which is almost identical to our estimate from equation (12) of 0.004 (0.075). In other words, including this interaction does not impact our estimate of ϕ_e.

Importantly, when we estimate the interaction term $\phi_{\pi,e}$, we find a small and insignificant value of 0.22 (0.28). As discussed in Section II.A, the large decline in inflation expectations did not lead to a noticeable decline in nominal earnings growth expectations. If we include this interaction term, the fundamental price-earnings ratio would be

$$pe_t^{fun} = \frac{1}{1-\rho\phi_e} E_t^*\left[\Delta e_{t+1}\right] - \frac{1}{1-\rho\phi_\pi}\left(1 - \frac{\rho\phi_{\pi,e}}{1-\rho\phi_e}\right) E_t^*\left[\pi_{t+1}\right]. \quad \text{(A2)}$$

Note that this is identical to equation (15), except for the $\left(1 - \frac{\rho\phi_{\pi,e}}{1-\rho\phi_e}\right)$ scaling on inflation expectations. Given that $\phi_{\pi,e}$ and ϕ_e are both near 0 and insignificant, this scaling term is close to 1 at 0.79 and is not statistically significantly different from 1.

Because the scaling term is near 1, this interaction term has almost no impact on the asset pricing results and by extension the return predictability results. Regressing the observed price-earnings ratio on the fundamental price-earnings ratio from equation (A2) produces a coefficient of almost exactly 1 at 1.07 (0.11) with an R^2 of 0.81. This is virtually identical to the results of Table V where the regression coefficient is 0.96 (0.09) and the R^2 is 0.81.

Regressing future 10-year nominal returns on the fundamental price-earnings ratio from (A2) produces a coefficient of -0.85 (0.07) with an R^2 of 0.39. Similarly, regressing 10-year real returns on the fundamental price-earnings ratio from (A2) produces a coefficient of -0.59 (0.12) with an R^2 of 0.24. These are almost identical to the results in Table VI.

D. *Connection to other models of expectation formation*

D.1. Believed persistence and response to surprises

To reduce repetition, let x be a general variable representing both inflation and nominal earnings growth. First, we show that under diagnostic expectations and sticky expectations, the believed persistence would need to equal the objective persistence. As shown in Bordalo, Gennaioli, and Shleifer (2018), for an objectively AR(1) variable, diagnostic expectations follow

$$\begin{aligned} E_t^*\left[x_{t+j}\right] &= E_t\left[x_{t+j}\right] + \theta\left(E_t\left[x_{t+j}\right] - E_{t-1}\left[x_{t+j}\right]\right) \\ &= \phi_x^j\left[x_t + \theta\left(x_t - \phi_x x_{t-1}\right)\right] \end{aligned} \tag{A1}$$

where θ is the parameter controlling the degree of representativeness bias. From equation (A1), the relationship between short-term and long-term expectations is

$$E_t^*\left[x_{t+1+j}\right] = \phi_x^j E_t^*\left[x_{t+1}\right]. \tag{A2}$$

In other words, the believed persistence equals the objective persistence.

Under sticky information, subjective expectations are

$$\begin{aligned} E_t^*\left[x_{t+j}\right] &= \lambda E_{t-1}^*\left[x_{t+j}\right] + (1-\lambda) E_t\left[x_{t+j}\right] \\ &= (1-\lambda)\sum_{k=0}^{\infty}\lambda^k E_{t-k}\left[x_{t+j}\right] \\ &= \phi_x^j (1-\lambda)\sum_{k=0}^{\infty}\left(\lambda\phi_x\right)^k x_{t-k} \end{aligned} \tag{A3}$$

where $1-\lambda$ represents the probability of updating information. From equation (A3), the relationship between long-term and short-term expectations is

$$E_t^*\left[x_{t+1+j}\right] = \phi^j E_t^*\left[x_{t+1}\right]. \tag{A4}$$

Once again, the believed persistence is equal to the objective persistence.

Second, we show that a model that only features bias in the persistence of variables would not allow short-term nominal earnings growth expectations to depend on recent surprises. Consider a model where agents believe the persistence of the variable is ϕ_x^* rather than ϕ_x. In this model, subjective expectations would be

$$\begin{aligned} E_t^*\left[x_{t+1}\right] &= \phi_x^* x_t \\ E_t^*\left[x_{t+1+j}\right] &= \phi_x^{*j} E_t^*\left[x_{t+1}\right]. \end{aligned}$$

By design, this model would allow the believed persistence of inflation to match the values we estimate from the survey data (ϕ_π^*, ϕ_e^*) rather than requiring that the believed persistence matches the objective persistence. For nominal earnings growth, survey short-term and long-term expectations imply a believed persistence of nearly 0, $\phi_e^* = 0.004$. In this model, this low believed persistence would imply that short-term expectations are virtually constant. Thus, this model would not match the large response of survey short-term nominal earnings growth expectations to surprises.

D.2. Fundamental extrapolation nests sticky information and adaptive expectations

Sticky information and adaptive expectations are both special cases of the fundamental extrapolation model where ϕ_x^* equals $\phi_x, 1$ respectively. Under sticky information, short-term expectations are

$$\begin{aligned} E_t^*[x_{t+1}] &= \lambda E_{t-1}^*[x_{t+1}] + (1-\lambda) E_t[x_{t+1}] \\ &= \lambda E_{t-1}^*[x_{t+1}] + (1-\lambda)\phi_x x_t. \end{aligned} \tag{A5}$$

As shown in Appendix D.1, long-term expectations are

$$E_t^*[x_{t+1+j}] = \phi_x^j E_t^*[x_{t+1}]. \tag{A6}$$

Combining equations (A5) and (A6), gives

$$E_t^*[x_{t+1}] = \phi_x x_t + \lambda\phi_x \left(E_{t-1}^*[x_t] - x_t\right). \tag{A7}$$

The description of short-term and long-term expectations given by equations (A6)-(A7) is identical to the fundamental extrapolation model equations (21)-(22) when $\phi_x^* = \phi_x$ and $\beta = \lambda\phi_x$. Intuitively, rational expectations at time t only depend on the realization x_t. Sticky expectations are a weighted sum of current and past rational expectations, so sticky expectations depend on a weighted sum of current and past realizations. This can also be seen directly in equation (A3).

Under adaptive expectations, subjective expectations are

$$E_t^*[x_{t+j}] = E_{t-1}^*[x_{t+j}] + \gamma\left(x_t - E_{t-1}^*[x_t]\right) \tag{A8}$$

where γ is the constant gain coefficient. This is equivalent to

$$E_t^*[x_{t+1}] = x_t + (1-\gamma)\left(E_{t-1}^*[x_t] - x_t\right) \tag{A9}$$

$$E_t^*[x_{t+j}] = E_t^*[x_{t+1}]. \tag{A10}$$

Equations (A9)-(A10) are identical to the fundamental extrapolation model equations (21)-(22) when $\phi_x^* = 1$ and $\beta = 1-\gamma$. Expectations that adapt quickly, i.e. high γ, translate to a low weight β for fundamental extrapolation as expectations mainly depend on recent realizations rather than older realizations.