

# What Drives Variation in the U.S. Debt/Output Ratio? The Dogs that Didn't Bark

Zhengyang Jiang<sup>1</sup>   Hanno Lustig<sup>2</sup>  
Stijn Van Nieuwerburgh<sup>3</sup>   Mindy Zhang Xiaolan<sup>4</sup>

<sup>1</sup>Northwestern Kellogg

<sup>2</sup>Stanford GSB

<sup>3</sup>Columbia Business School

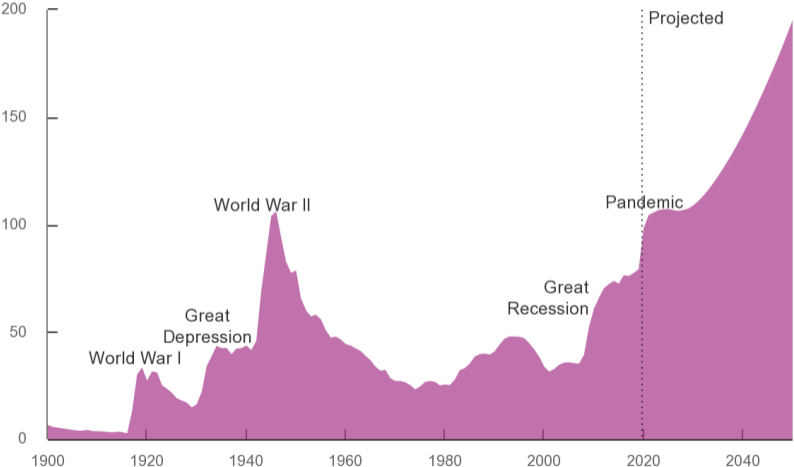
<sup>4</sup>University of Texas at Austin

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# Fiscal Sustainability

## Federal Debt Held by the Public, 1900 to 2050

Percentage of Gross Domestic Product



Source: Congressional Budget Office

# Fiscal Sustainability: Forward-looking Approach

- ▶ Ongoing debate in the U.S. about fiscal sustainability
- ▶ Current run-up in the U.S. debt/output ratio reflects:
  1. Lower future inflation-and-growth adjusted returns on government debt (Blanchard, 2019; Furman and Summers, 2020; Cochrane, 2021a) :
    - ▶  $(r - g) < 0$  debate
  2. Higher future surpluses (Bohn, 1998; Cochrane, 2020)
  3. Higher future debt/output ratio

# This Paper

- ▶ Apply standard asset pricing machinery (**Campbell-Shiller** decomposition) to a macro question (fiscal sustainability)
  - ▶ Campbell-Shiller decomposition of the U.S. debt/output ratio :
    1. **Discount rates:** No evidence that the debt/output ratio predicts real growth-adjusted returns. ✗
    2. **Cash flows:** No evidence that the debt/output ratio predicts surpluses. ✗
    3. **Residual:** higher future debt/output ratio ✓
- ⇒ Excess smoothness: Bond prices today not responsive to news about future macro fundamentals

# Findings Differ From Literature

- ▶ Earlier work:
  - ▶ [Bohn \(1998\)](#), studying a sample that ends in the mid-1990s, finds evidence that the primary surplus increases when the debt/output ratio is high
  - ▶ [Cochrane \(2021a,b\)](#) finds evidence that the debt/output ratio predicts lower *nominal* returns on the government debt portfolio
  - ▶ This paper: no evidence that the debt/output ratio predicts surpluses or *real growth-adjusted* returns
- ▶ **Key observation:** Large small-sample bias ([Stambaugh, 1999](#)) in the slope coefficients of the return and surplus predictability regressions due to:
  1. High **persistence** of the debt/output ratio (the predictor)
  2. High **correlation** between the innovations to the predictor and the predicted variables

# Related Literature

- ▶ **Stock return predictability** (Campbell and Thompson, 2007; Cochrane, 2008; Binsbergen and Koijen, 2010; Goyal and Welch, 2005; Golez and Koudijs, 2018):
  - ▶ Discount rates on stocks are remarkably volatile (Hansen and Jagannathan, 1991),
  - ▶ Valuation of stocks seems excessively volatile compared to its fundamentals (LeRoy and Porter, 1981; Shiller, 1981),
  - ▶ High valuations imply low future returns (mean reversion in valuation ratios),
- ▶ **Bond return predictability:** (Fama and Bliss, 1987; Campbell and Shiller, 1991; Cochrane and Piazzesi, 2005; Ludvigson and Ng, 2009; Cochrane, 2011) ,
  - ▶ Individual bond return predictability,
  - ▶ For entire bond portfolio: high valuations do not imply low future returns (no mean reversion in valuation ratios),
  - ▶ Valuation of bonds seems too smooth compared to its fundamentals

# Variance Decomposition of Debt/Output

# Campbell-Shiller Decomposition of Debt/Output Ratio

- ▶ Log-linearized return equation implied by the government budget constraint:

$$\tilde{r}_{t+1} = r_{t+1} - \pi_{t+1} - x_{t+1} = \rho v_{t+1} - v_t + s_{t+1},$$

where  $\rho = \exp(-(r - x - \pi))$  is a constant,  $v_t$  is log of debt/output ratio, and  $s_{t+j} = sy_{t+j}/e^v$  is a scaled measure of surplus/output.

(see Gourinchas and Rey, 2007; Berndt, Lustig, and Yeltekin, 2012; Cochrane, 2021a)

- ▶ Similar to log-linearized return for stocks:

$$r_{t+1} = \rho pd_{t+1} - pd_t + \Delta d_{t+1}.$$

- ▶ By iterating this forward  $T$  times and taking expectations, we obtain the debt valuation equation:

$$v_t = \mathbb{E}_t \sum_{j=1}^T \rho^{j-1} (s_{t+j} - \tilde{r}_{t+j}) + \mathbb{E}_t \rho^T v_{t+T}.$$



# Variance Decomposition

- ▶ We set  $\rho = 1$  (“ $r=g$ ”).
- ▶ Debt/output ratio reflects either future surpluses or future returns after adjusting for inflation and growth.

$$v_t = \mathbb{E}_t \sum_{j=1}^T (s_{t+j} - \tilde{r}_{t+j}) + \mathbb{E}_t v_{t+T}.$$

- ▶ Debt/output ratio varies because it either predicts future surpluses, future returns, or the future debt/output ratio:

Variance Decomposition of the Debt/Output Ratio.

$$\text{var}(v_t) = \text{cov} \left( \sum_{j=1}^T s_{t+j}, v_t \right) - \text{cov} \left( \sum_{j=1}^T \tilde{r}_{t+j}, v_t \right) + \text{cov}(v_t, v_{t+T}).$$

# Variance Decomposition: Implementation

- ▶ Estimate a system of univariate forecasting regressions for  $\sum_{j=1}^T s_{t+j}$ ,  $\sum_{j=1}^T \tilde{r}_{t+j}$ ,  $v_{t+j}$  using the lagged debt/output ratio as a predictor:

$$\begin{aligned}\sum_{j=1}^T s_{t+j} &= a_s + b_T^s v_t + \epsilon_{t+T}^s, \\ \sum_{j=1}^T \tilde{r}_{t+j} &= a_r + b_T^r v_t + \epsilon_{t+T}^r, \\ v_{t+T} &= \phi_0 + \phi_T v_t + \epsilon_{t+T}^v.\end{aligned}$$

- ▶ More reliable estimates of long-run dynamics than VAR ([Jordà, 2005](#))
- ▶ [Cochrane \(2008\)](#); [Lettau and Van Nieuwerburgh \(2008\)](#) adopt the same approach to implementing a Campbell-Shiller decomposition of the price/dividend ratio for stocks.

# Variance Decomposition: Implementation

- ▶ Regression coefficients can be interpreted as the fraction of the variance of  $v_t$  explained by each component for a certain horizon  $T$ :

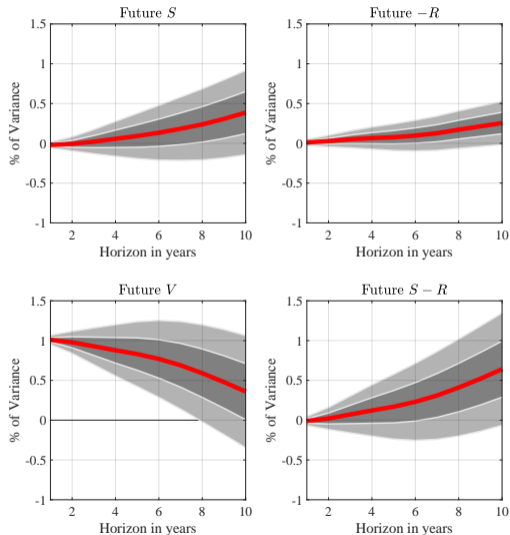
$$\frac{\text{cov}(\sum_{j=1}^T s_{t+j}, v_t)}{\text{var}(v_t)} = b_T^s,$$
$$\frac{\text{cov}(-\sum_{j=1}^T \tilde{r}_{t+j}, v_t)}{\text{var}(v_t)} = -b_T^r,$$
$$\frac{\text{cov}(v_{t+T}, v_t)}{\text{var}(v_t)} = \phi_T.$$

- ▶ Cross-equation restriction is satisfied:  $b_T^s - b_T^r + \phi_T = 1, \forall T$ .
- ▶ **Fiscal sustainability:**  $\phi_T < 1$  for all  $T$  and  $\phi_T \rightarrow 0$  as  $T \rightarrow \infty$ .

# Variance Decomposition of $v_t$ : No Bias Correction (1947-2020)

<i>Horizon</i>	1	2	3	4	5	6	7	8	9	10
<i>Forecasting <math>\sum_{j=1}^T -\tilde{r}_{t+j}</math></i>										
$-b_T^r$	0.01	0.03	0.05	0.07	0.08	0.1	0.13	0.17	0.21	0.25
<i>s.e.</i>	0.02	0.04	0.05	0.07	0.08	0.09	0.11	0.12	0.13	0.13
$R^2$	0.01	0.02	0.03	0.04	0.04	0.05	0.06	0.08	0.10	0.12
<i>Forecasting <math>\sum_{j=1}^T s_{t+j}</math></i>										
$b_T^s$	-0.02	-0.01	0.02	0.06	0.09	0.13	0.18	0.24	0.31	0.39
<i>s.e.</i>	0.02	0.04	0.08	0.11	0.14	0.17	0.2	0.22	0.24	0.26
$R^2$	0.02	0	0	0.01	0.02	0.03	0.05	0.06	0.09	0.11
<i>Forecasting <math>v_{t+T}</math></i>										
$\phi$	1.01	0.98	0.93	0.88	0.83	0.77	0.69	0.59	0.48	0.36
<i>s.e.</i>	0.03	0.07	0.11	0.16	0.2	0.24	0.27	0.3	0.33	0.35
$R^2$	0.95	0.85	0.74	0.64	0.54	0.43	0.32	0.22	0.13	0.07

# Variance Decomposition of $v_t$ : No Bias Correction (1947-2020)



- ▶ Cannot reject the null of the presence of the unit root
- ▶ At the 5-yr horizon, 83% of the debt/output fluctuations can be attributed to the future debt/output
- ▶ At the 10-hr horizon, both cash flow and discount rate channels start to matter, but cannot reject the null that the fraction is zero

# Small-sample Bias in Predictive Coefficients

- ▶ Small-sample bias [Stambaugh \(1999\)](#); [Boudoukh, Israel, and Richardson \(2020\)](#) for horizon  $T$ :

$$\begin{aligned} bias_T^r &= \mathbb{E} \left( \widehat{b}_T^r - b_T^r \right) = \frac{1}{N} \left[ T(1 + \phi) + 2\phi \frac{1 - \phi^T}{1 - \phi} \right] \times -\frac{cov(\epsilon^v, \epsilon^r)}{var(\epsilon^v)}, \\ bias_T^s &= \mathbb{E} \left( \widehat{b}_T^s - b_T^s \right) = \frac{1}{N} \left[ T(1 + \phi) + 2\phi \frac{1 - \phi^T}{1 - \phi} \right] \times -\frac{cov(\epsilon^v, \epsilon^s)}{var(\epsilon^v)}, \end{aligned}$$

where  $\phi$  is first-order autocorrelation of  $v_t$ ,  $N$  sample size

- ▶ Here:  $\phi = .99$ ,  $corr(\epsilon^v, -\epsilon^r) = -0.75$  and  $corr(\epsilon^v, \epsilon^s) = -0.85$ .

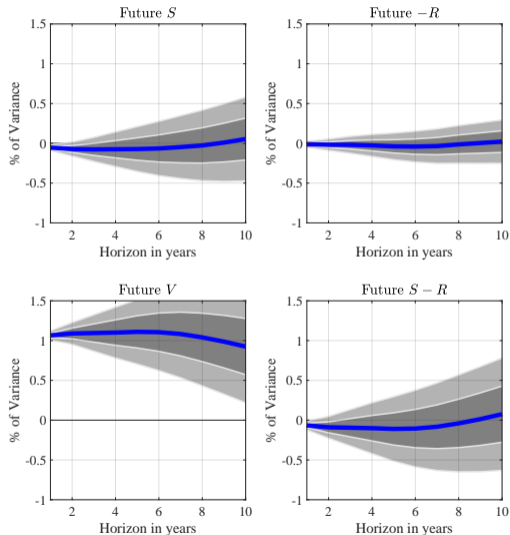
⇒ Biases for  $b_T^s$  and  $-b_T^r$  are positive and large.

⇒ We are overstating the surplus and return predictability in small samples.

# Variance Decomposition of $v_t$ : Bias Correction (1947-2020)

Horizon	1	2	3	4	5	6	7	8	9	10
<i>Forecasting <math>\sum_{j=1}^T -\tilde{r}_{t+j}</math></i>										
$-b_T^r$	0.01	0.03	0.05	0.07	0.08	0.1	0.13	0.17	0.21	0.25
s.e.	0.02	0.04	0.05	0.07	0.08	0.09	0.11	0.12	0.13	0.13
$R^2$	0.01	0.02	0.03	0.04	0.04	0.05	0.06	0.08	0.1	0.12
<i>unbiased</i>	<b>-0.01</b>	<b>-0.02</b>	<b>-0.02</b>	<b>-0.03</b>	<b>-0.04</b>	<b>-0.04</b>	<b>-0.04</b>	<b>-0.01</b>	<b>0</b>	<b>0.02</b>
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<i>Forecasting <math>v_{t+T}</math></i>										
$\phi$	1.01	0.98	0.93	0.88	0.83	0.77	0.69	0.59	0.48	0.36
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$R^2$	0.95	0.85	0.74	0.64	0.54	0.43	0.32	0.22	0.13	0.07
<i>unbiased</i>	<b>1.07</b>	<b>1.09</b>	<b>1.1</b>	<b>1.1</b>	<b>1.11</b>	<b>1.11</b>	<b>1.08</b>	<b>1.04</b>	<b>0.99</b>	<b>0.92</b>

# Variance Decomposition of $v_t$ : Bias Correction (1947-2020)



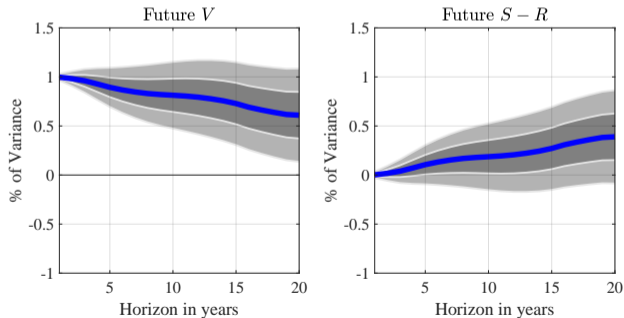
- ▶ The bias-corrected variance decomposition attributes  $-4\%$  and  $-7\%$  of the debt/output ratio variance to the discount rate and cash flow channel respectively at the 5-year horizon.
- ▶ As a result, 111% is accounted for by the future debt/output ratio at the 5-year horizon.
- ▶ At the 10-year horizon, we still attribute 92% of the variance to the future debt/output ratio, after correcting for the small-sample bias.



# Variance Decomposition: Robustness

- ▶ Longer U.S. Hall-Payne-Sargent sample: 1842–2020
  - ▶ Same conclusion after small-sample bias correction
  - ▶ Now have more power to reject the null of no return predictability

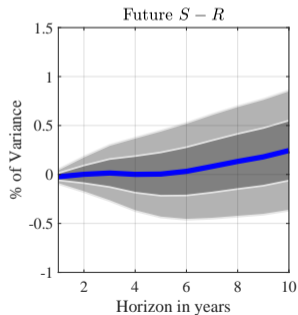
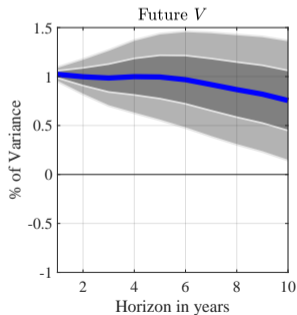
*After Bias Correction*



# Variance Decomposition: Robustness

- ▶ Longer U.S. Hall-Payne-Sargent sample: 1842–2020 ✓
- ▶ Shorter Bohn Sample 1948—1995 ✓

*After Bias Correction*

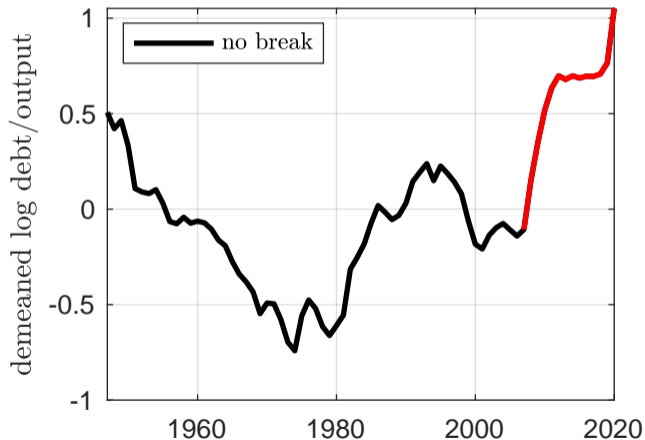


# Permanent Shocks to the Debt/Output Ratio

1. Simulation under Null of Unit Root simulation
2. Structural Break

## Structural Breaks

- ▶ A major contributor to the small role of fundamentals is the large run-up in debt/output ratio during the GFC



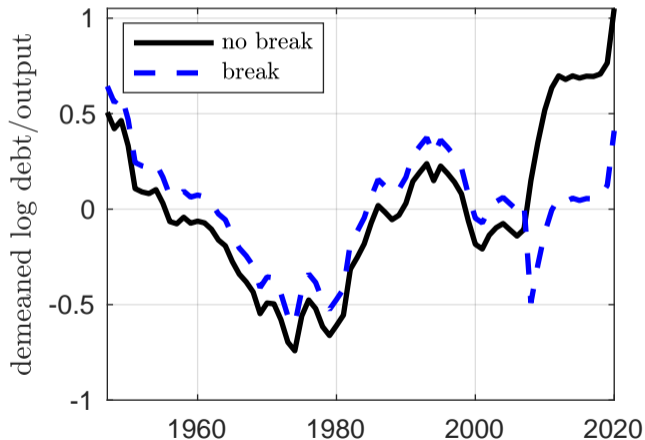
## Structural Breaks

- ▶ A major contributor to the small role of fundamentals is the large run-up in debt/output ratio during the GFC
- ▶ Structural break in the log debt/output ratio ( [Lettau and Van Nieuwerburgh \(2008\)](#)): demean the log debt/output ratio  $\tilde{v}_t = v_t - \bar{v}_t$  with a lower pre-2007 sample mean ( $\bar{v}_t, t < 2007$ ) and a higher post-2007 sample mean ( $\bar{v}_t, t \geq 2007$ ).
  - ▶ This structural break introduces a 78 log point permanent increase in the debt/output ratio; we 'delete' this increase from the variance decomposition.
  - ▶ Decrease in  $\phi$  has to increase surplus/return predictability (cross-equation restriction):  $(b_T^s - b_T^r) \nearrow = (1 - \phi_T) \nearrow$ .
- ▶ Variance of the **transitory component** of debt/output ratio:

$$\text{var}(\tilde{v}_t) = \text{cov} \left( \sum_{j=1}^T s_{t+j}, \tilde{v}_t \right) - \text{cov} \left( \sum_{j=1}^T \tilde{r}_{t+j}, \tilde{v}_t \right) + \text{cov}(\tilde{v}_t, \tilde{v}_{t+T}).$$

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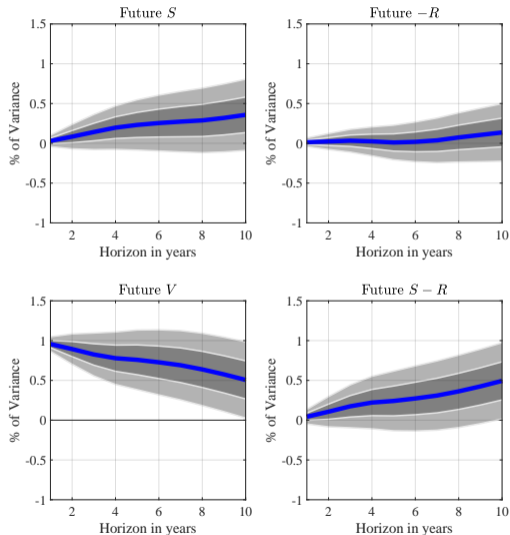


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# Variance Decomposition of $\tilde{v}_t$ with Break



- ▶ Stronger evidence for surplus, but not return predictability
- ▶ Fundamentals now account for about 50% of the variation in the **transitory component** of the debt/output ratio at the 10-year horizon
- ▶ Still leave the large, permanent increase in the debt/output ratio (as well as its timing) unexplained



## Structural Break Candidate 1: Biased Beliefs

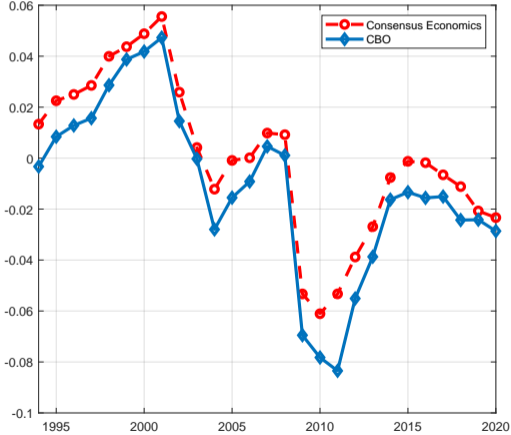
- ▶ Econometrician does not predict higher surpluses or lower returns when the debt/output ratio rises, but bond investors may.
- ▶ If investors systematically *over-predict* surpluses and *under-predict* returns when the debt/output ratio increases, their forecast error can impute a unit root in the debt/output ratio under the actual measure  $\mathbb{E}$ , while the debt/output ratio is stationary under the subjective beliefs measure  $\mathbb{F}$

$$v_t = \mathbb{E}_t \sum_{j=1}^T (s_{t+j} - \tilde{r}_{t+j}) + \underbrace{\left( \mathbb{F}_t v_{t+T} + \overbrace{(\mathbb{F}_t - \mathbb{E}_t) \sum_{j=1}^T (s_{t+j} - \tilde{r}_{t+j})}^{\text{ForcErr}} \right)}_{\mathbb{E}_t v_T},$$

- ▶  $Cov(v_t, \mathbb{E}_t v_T)$  large and  $Cov(v_t, \mathbb{F}_t v_T)$  small if  $Cov(v_t, \text{ForcErr}) \gg 0$

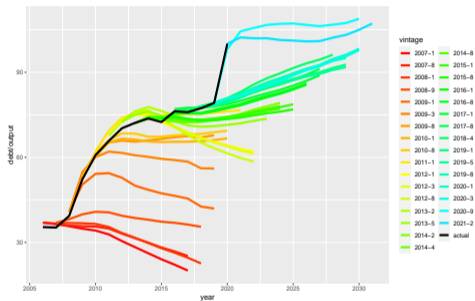
# Private Forecasts Align with CBO Forecasts

Figure: Comparing CBO and Private-Sector Surplus Forecasts

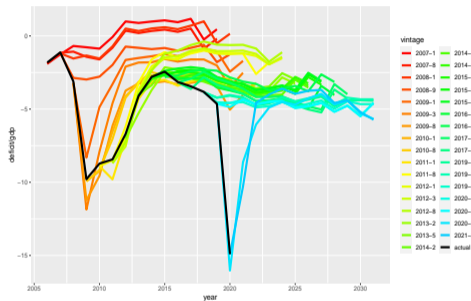


# Ten-year CBO Projections

*Debt/Output*



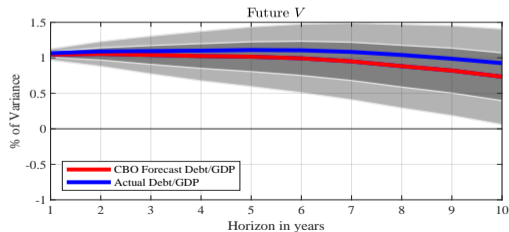
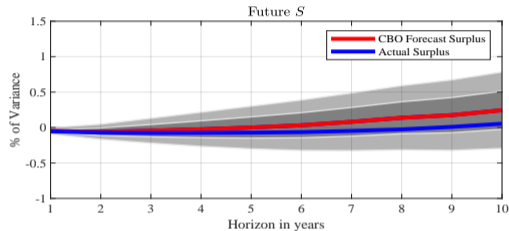
*Surplus/Output*



- ▶ CBO systematically over-predicts future surpluses when debt rises and underpredicts future debt/output, especially since GFC.
- ▶ Forecast errors were close to zero from 1980 to 1997.

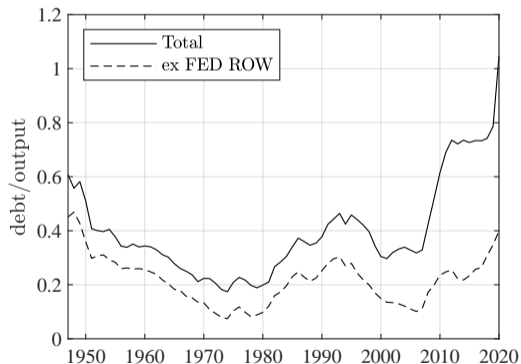
# Predictability Under Subjective Measure

- ▶ Estimate the CS decomposition under subjective beliefs
  - ▶ Using the CBO forecast for the surplus/GDP ratio after 2007.



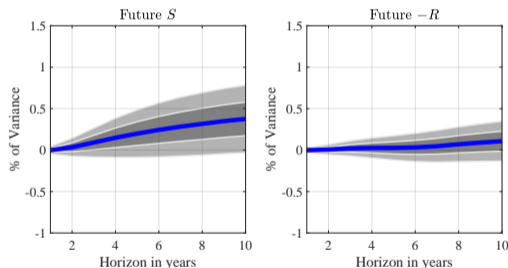
## Structural Break Candidate 2: Fed & ROW

- ▶ Fed and Foreign holdings of Treasuries accelerated after GFC (QE)
- ▶ Private domestic holdings (ex-Fed, ex-ROW) are candidate transitory component  $\tilde{v}_t$



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- ▶ We still cannot reject the null that surpluses are not predictable

# Conclusion

- ▶ The U.S. bond market's valuation surprisingly insensitive to news about future surpluses or returns
- ▶ Difficult to reject null hypothesis of unit root in debt/output once small-sample bias is addressed
- ▶ Interpretations: persistent component in debt/output ratio (structural break after 2007) imputed by
  1. Fed and ROW purchases
  2. Bond market investors' (overly optimistic) beliefs about future fiscal rectitude

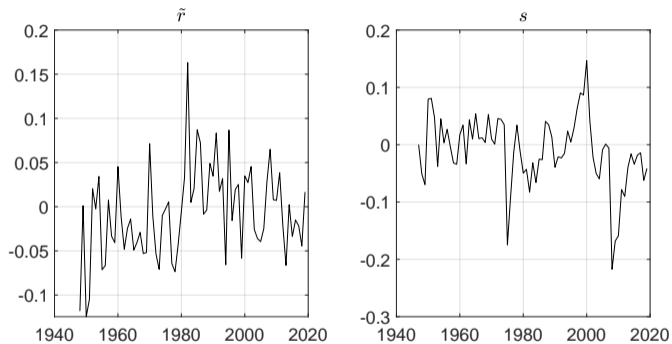
## Data: Decade Averages

	$\tilde{r}$	$r$	$x$	$\pi$	$x + \pi$	$s/y$
1947-1949	-7.8%	-1.8%	0.6%	5.4%	6.0%	1.5%
1950-1959	-3.8%	2.7%	4.1%	2.4%	6.5%	1.4%
1960-1969	-2.8%	3.9%	4.4%	2.3%	6.7%	1.4%
1970-1979	-2.5%	7.0%	3.2%	6.3%	9.5%	-0.6%
1947-1979	-3.5%	3.9%	3.6%	3.8%	7.4%	0.8%
1980-1989	4.1%	11.8%	3.0%	4.6%	7.6%	0.1%
1990-1999	1.6%	6.9%	3.2%	2.2%	5.3%	1.5%
2000-2009	0.8%	4.9%	1.9%	2.2%	4.1%	0.0%
2010-2020	-0.4%	2.9%	1.7%	1.6%	3.3%	-0.4%
1980-2020	1.5%	6.5%	2.4%	2.6%	5.1%	-0.6%
1947-2020	-0.7%	5.4%	3.0%	3.2%	6.1%	0.1%

- ▶ Note that  $r < g$  or  $\tilde{r} < 0$  only in first half of post-war sample
- ▶ Surpluses came down over time
- ▶ Does variation in  $v_t$  predict this secular variation in  $\tilde{r}_{t \rightarrow t+10}$  or  $s_{t \rightarrow t+10}$ ?



# Returns and Surpluses

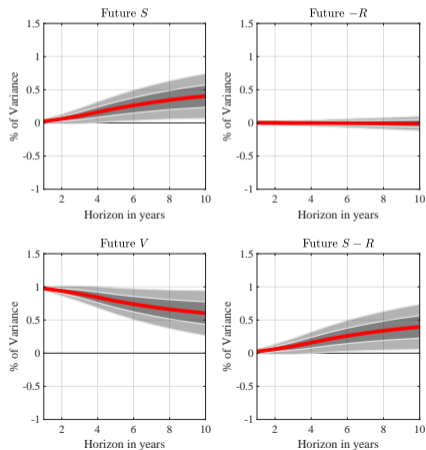


This figure plots the inflation-and-growth-adjusted log returns  $\tilde{r}_t$  and the surplus/output ratio  $s_t$ .

# Variance Decomposition of $v_t$ : Longer Sample 1842—2020

- ▶ Robustness to longer U.S. Hall-Payne-Sargent sample

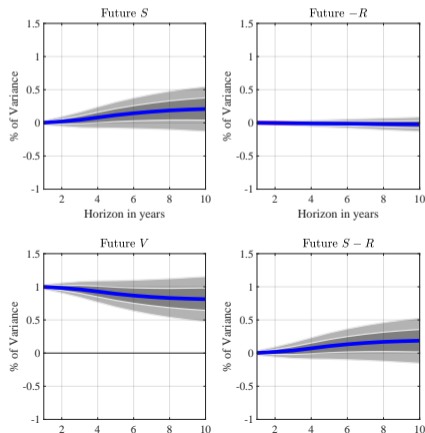
*Panel A: Before Bias Correction*



# Variance Decomposition of $v_t$ : Longer Sample 1842—2020

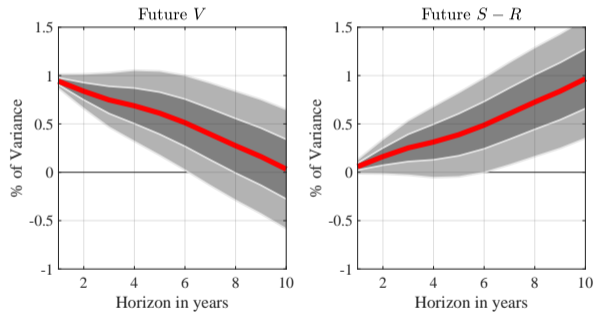
- ▶ Same conclusion after small-sample bias correction
- ▶ Now have more power to reject the null of no return predictability

*Panel B: After Bias Correction*



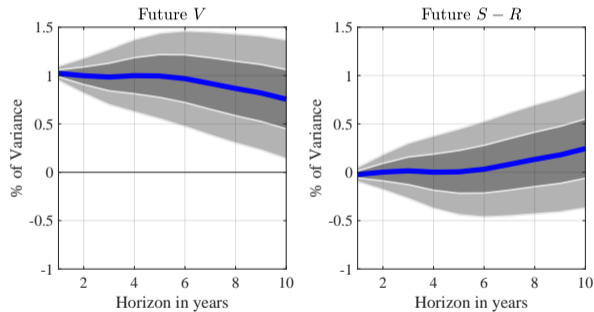
# Variance Decomposition of $v_t$ : Shorter Bohn Sample 1948—1995

*Panel A: Before Bias Correction*



# Variance Decomposition of $v_t$ : Shorter Bohn Sample 1948—1995

*Panel B: After Bias Correction*



# Forecasting Nominal Returns and Inflation with $v_t$

<i>Horizon</i>	1	2	3	4	5	6	7	8	9	10
<i>Forecasting <math>\sum_{j=1}^T r_{t+j}</math></i>										
$b_T^r$	-0.05	-0.11	-0.16	-0.22	-0.28	-0.35	-0.43	-0.52	-0.6	-0.69
<i>s.e.</i>	[0.02]	[0.03]	[0.05]	[0.06]	[0.07]	[0.08]	[0.09]	[0.1]	[0.11]	[0.13]
<i>Forecasting <math>\sum_{j=1}^T x_{t+j}</math></i>										
$b_T^x$	0	0	0.01	0	0	0	0.01	0.01	0.02	0.03
<i>s.e.</i>	[0.01]	[0.02]	[0.03]	[0.04]	[0.05]	[0.06]	[0.06]	[0.07]	[0.08]	[0.08]
<i>Forecasting <math>\sum_{j=1}^T \pi_{t+j}</math></i>										
$b_T^\pi$	-0.04	-0.08	-0.12	-0.16	-0.21	-0.26	-0.31	-0.37	-0.42	-0.48
<i>s.e.</i>	[0.01]	[0.01]	[0.02]	[0.02]	[0.03]	[0.04]	[0.05]	[0.06]	[0.08]	[0.09]
<i>Forecasting <math>\sum_{j=1}^T \tilde{r}_{t+j}</math></i>										
$b_T^{\tilde{r}}$	-0.01	-0.03	-0.05	-0.06	-0.07	-0.09	-0.13	-0.16	-0.2	-0.25
<i>s.e.</i>	[0.01]	[0.02]	[0.03]	[0.04]	[0.05]	[0.06]	[0.06]	[0.07]	[0.08]	[0.09]

back

# Forecasting Returns and Surpluses with $\tilde{v}_t$

Horizon	1	2	3	4	5	6	7	8	9	10
Structural Break										
Forecasting $\sum_{j=1}^T -\tilde{r}_{t+j}$										
$-b_T^r$	0.03	0.05	0.07	0.07	0.07	0.08	0.11	0.16	0.2	0.24
s.e.	0.03	0.05	0.07	0.09	0.11	0.13	0.14	0.16	0.17	0.18
$R^2$	0.02	0.03	0.04	0.03	0.02	0.02	0.04	0.06	0.08	0.1
<i>unbiased</i>	0.01	0.02	0.03	0.02	0.01	0.02	0.04	0.07	0.11	0.14
Forecasting $\sum_{j=1}^T s_{t+j}$										
$b_T^s$	0.07	0.16	0.25	0.34	0.41	0.46	0.51	0.56	0.62	0.68
s.e.	0.03	0.07	0.11	0.13	0.16	0.17	0.19	0.2	0.21	0.23
$R^2$	0.04	0.12	0.2	0.29	0.36	0.42	0.47	0.5	0.53	0.57
<i>unbiased</i>	0.03	0.08	0.14	0.2	0.23	0.25	0.27	0.29	0.32	0.36
Forecasting $v_{t+T}$										
$\phi$	0.91	0.79	0.68	0.59	0.53	0.45	0.38	0.29	0.19	0.08
s.e.	0.05	0.09	0.13	0.16	0.19	0.2	0.22	0.23	0.23	0.24
$R^2$	0.86	0.7	0.55	0.44	0.35	0.27	0.19	0.11	0.05	0.01
<i>unbiased</i>	0.96	0.89	0.83	0.78	0.76	0.73	0.69	0.64	0.58	0.51

## Simulation from Unit Root Model

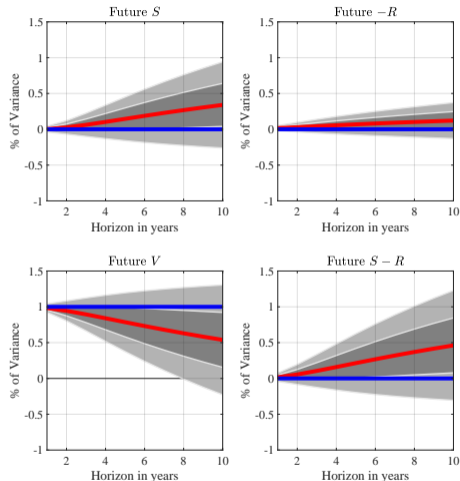
- ▶ Evidence is consistent with a unit root in the debt/output ratio.
- ▶ Simulate under the null that there is unit root in the debt/output ratio:

$$\begin{aligned}v_{t+1} &= v_t + \Delta v_{t+1} \\ \Delta v_{t+1} &= \psi_0 + \psi_1 \Delta v_t + \epsilon_{t+1}^v \\ \tilde{r}_{t+1} &= r_0 + \epsilon_{t+1}^r\end{aligned}$$

- ▶ Estimate  $(\epsilon_{t+1}^v, \epsilon_{t+1}^r)$  in historical data
  - ▶ Draw 10,000 samples of length  $N$  with replacement from observed  $(\epsilon_{t+1}^v, \epsilon_{t+1}^r)$
- ▶ There is no contribution from return/surplus predictability (fundamentals):  
 $b_T^s = b_T^r = 0 = 1 - \phi_T$  at all horizons  $T$ .
- ▶ Simulate and estimate predictability regressions on simulated data
- ▶ Evaluate accuracy of small-sample bias correction



# Variance Decomposition of $v_t$ under Unit Root

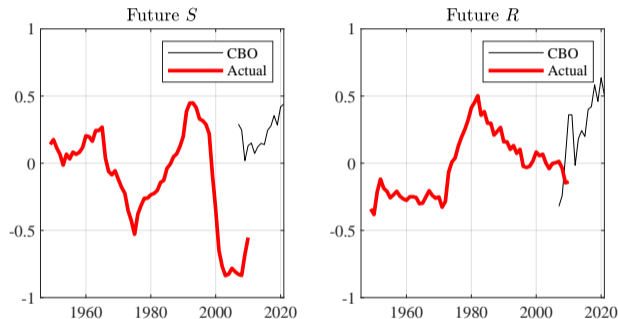


- ▶ The average slope coefficients obtained from the unit root model imply variance decomposition close to our point estimates in the case without bias correction.
- ▶ Spurious evidence of mean reversion that creates a large role for fundamentals over longer horizons, in cases where there is no mean-reversion.

The mean of the small-sample slope coefficients in red; the long-sample slope coefficients in blue

# CBO Projections vs. Realized

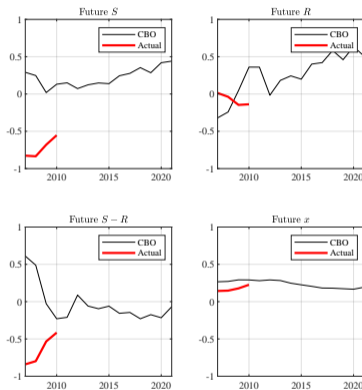
$$v_t = \mathbb{F}_t \sum_{j=1}^{10} (s_{t+j} - \tilde{r}_{t+j}) + \mathbb{F}_t v_{t+10}$$



Decomposition of the log debt/output ratio  $v_t$  into components due to CBO-projected (and realized) future government surpluses  $\sum_{j=1}^T s_{t+j}$ , future discount rates  $\sum_{j=1}^T \tilde{r}_{t+k}$ , for  $T = 10$ .

# CBO Projections vs. Realized

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# Related Literature

- ▶ **Statistical issues with persistent predictors** (Nelson and Kim, 1993; Hamilton, 1994; Stambaugh, 1999; Lewellen, 2004; Torous, Valkanov, and Yan, 2004; Campbell and Yogo, 2006; Boudoukh, Israel, and Richardson, 2020; Bauer and Hamilton, 2017)
- ▶ **Fiscal policy and budget constraints:** Hansen, Roberds, and Sargent (1991); Hamilton and Flavin (1986); Trehan and Walsh (1988, 1991); Bohn (1998, 2007); D'Erasmus, Mendoza and Zhang (2016); Blanchard (2019); Barro (2020), Reis (2020), Brunnermeier, Merkel and Sannikov (2020), Jiang, Lustig, Van Nieuwerburgh and Xiaolan (2019, 2020, 2021a,b,c).
- ▶ **Safe asset supply:** Gourinchas and Rey (2007); Caballero, Farhi, and Gourinchas (2008); Caballero and Krishnamurthy (2009); Maggiori (2007); He, Krishnamurthy, and Milbradt (2018); Jiang, Krishnamurthy and Lustig (2018, 2019).

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