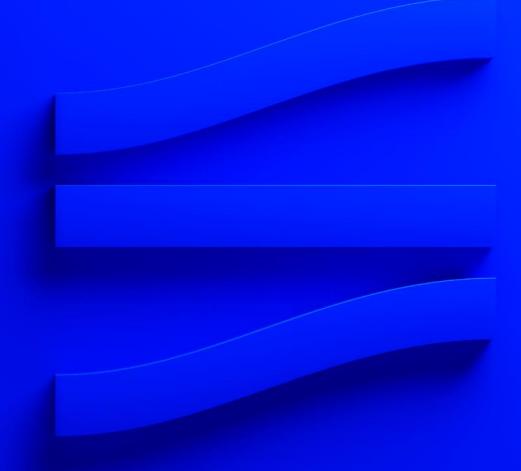


# Relevance-based prediction: A transparent and adaptive alternative to machine learning

David Turkington Co-authors: Megan Czasonis and Mark Kritzman



5940080.1.1.GBL.

### **Relevance-based prediction: Overview**

- **Relevance-based prediction** is a new forecasting routine that forms a prediction as a weighted average of observed outcomes that vary through time or cross-sectionally, in which the weights are based on a precise statistical measure called relevance.
  - Relevance has two components, similarity and informativeness, which are both measured as Mahalanobis distances.
- Relevance-based prediction depends crucially on fit, which measures the average alignment of relevance and outcomes across all the observations that go into a prediction.
  - Fit reveals the **confidence** one should assign to an individual prediction.
  - Fit also identifies the optimal combination of observations and predictive variables for each individual prediction task.

### Key advantages of relevance-based prediction

- Relevance-based prediction addresses complexities, such as asymmetry between predictive variables and outcomes, that are beyond the capacity of linear regression analysis.
- Relevance-based prediction is **more transparent and more adaptive** than modelbased machine learning algorithms, and it is **theoretically justified**.
- Relevance-based prediction is a sophisticated extension of model-free machine learning algorithms, that uniquely **converges to linear regression analysis** under special circumstances.





### The task: Predict Y given X

### **Model-based approach**

### Model-free approach

Posit a model (prediction function and set of parameters) that will be used for every prediction task.

Estimate the parameters based on prior observations, and then discard the observations.

For a *specific prediction task*, determine how much weight to place on each prior observation.

Extract as much information as needed from the pertinent observations.



### The task: Predict Y given X

Model-based approach

y = f(x)

estimate f(x) using X, Y

 $\hat{y}_t = f(x_t)$ 

### **EXAMPLES:**

Linear regression, Lasso Decision trees, random forest Neural networks, transformers

### Model-free approach

$$\hat{y}_t = \sum_i w_{it} y_i$$

choose  $w_{it} = g(x_t, X, Y)$ 

EXAMPLES: Nearest neighbors Kernel regression Relevance-based prediction



### Learning from new observations: Three methods

### 1. Retrain completely

Recompute the model's parameters from scratch using all data (including new data)

### 2. Online learning

Incrementally update the model's parameters at a defined "learning rate"

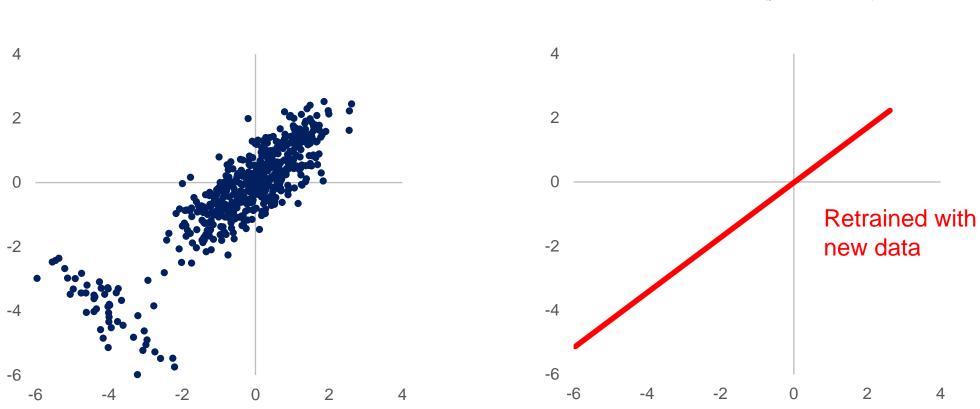
### 3. Model-free adaptive learning

Dynamically retrieve new data as needed for each prediction task



## **Example: (1) Complete retraining**

**Observed data** 



Model view ( $\beta = 0.86$ )

For illustration only. The original observations are 500 random draws of two variables with 0.8 correlation, and additional observations are 50 random draws of two variables with -0.9 correlation. The learning rate (alpha) equals 1/500. Source: State Street Global Markets.



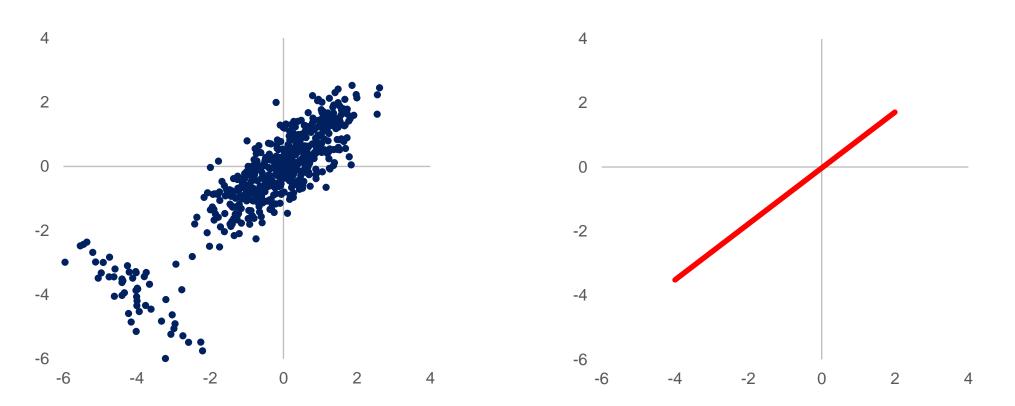
7

### **Example: (2) Online learning**

Update rule:  $\beta_{new} = \beta_{old} + \alpha (y_{\tau} - \hat{y}_{\tau}) x_{\tau}$ 

### Observed data



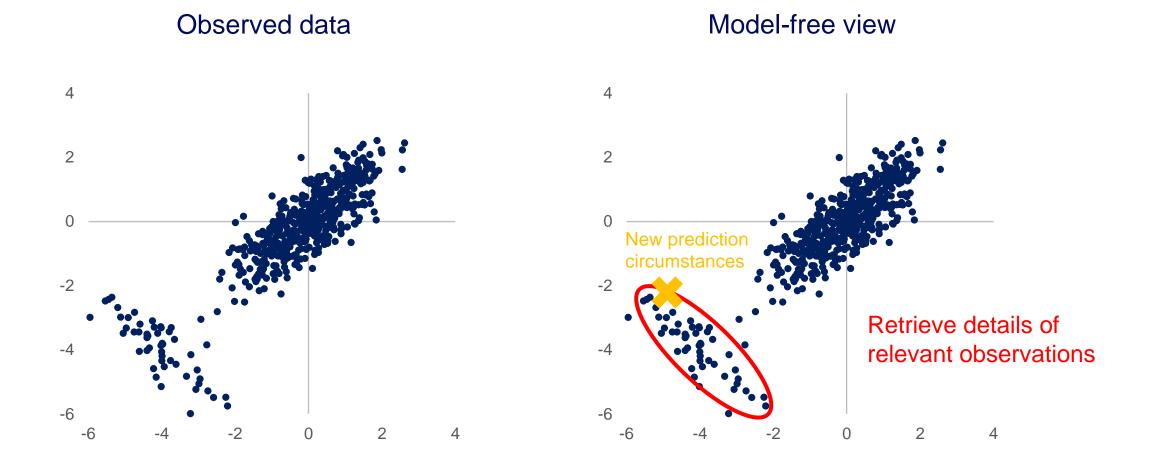


For illustration only. The original observations are 500 random draws of two variables with 0.8 correlation, and additional observations are 50 random draws of two variables with -0.9 correlation. The learning rate (alpha) equals 1/500. Source: State Street Global Markets.

### 5940080.1.1.GBL.



### **Example: (3) Model-free adaptive learning**



For illustration only. The original observations are 500 random draws of two variables with 0.8 correlation, and additional observations are 50 random draws of two variables with -0.9 correlation. The learning rate (alpha) equals 1/500. Source: State Street Global Markets.



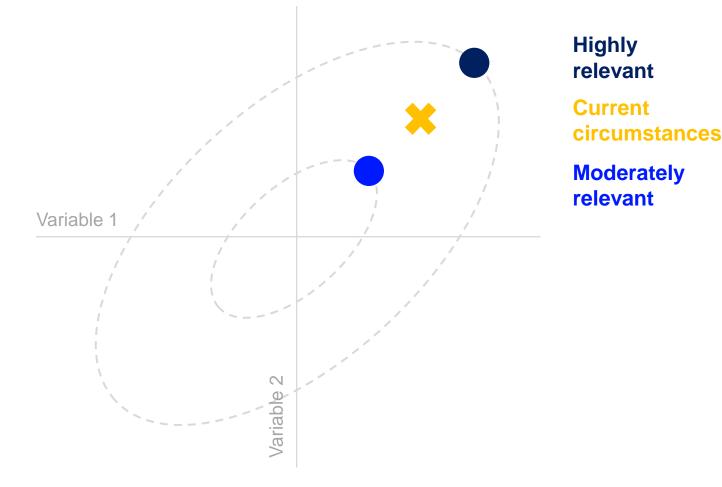
# Relevance





### What is a principled way to determine relevance?

- A past observation is relevant to current prediction circumstances to the extent that it is:
  - Similar to current circumstances, and
  - Different from average





### **Relevance: Precise multivariate definition**

The relevance of an observation  $(x_i)$  to a prediction circumstance  $(x_t)$  is:

$$r_{it} = sim(x_i, x_t) + \frac{1}{2} (info(x_i, \bar{x}) + info(x_t, \bar{x}))$$

$$sim(x_i, x_t) = -\frac{1}{2}(x_i - x_t)\Omega^{-1}(x_i - x_t)'$$

Observation's proximity to current circumstances

 $info(x_i, \bar{x}) = (x_i - \bar{x})\Omega^{-1}(x_i - \bar{x})'$ 

Observation's distance from average

 $info(x_t, \bar{x}) = (x_t - \bar{x})\Omega^{-1}(x_t - \bar{x})'$ 

Centers relevance around zero



## Why relevance is not arbitrary (Part 1: information theory)

• Information theory tells us that the information in an observation is the negative logarithm of its likelihood.

 $information(x_i) = -\log(p(x_i))$ 

Sum	Total possibilities	With this sum	Probability	Information (base 10)
100	10,000,000,000	1	0.000000001	10
99	10,000,000,000	10	0.000000010	9
98	10,000,000,000	55	0.000000055	8.26

. . .

## Why relevance is not arbitrary (Part 1: information theory)

- Information theory tells us that the information in an observation is the negative logarithm of its likelihood.
- It also tells us that the normal distribution has **maximum entropy** for a given variance.
- In addition, the **Central Limit Theorem** motivates the normal distribution as a common limiting case of many random processes.
- The **Mahalanobis distance** measures the information in a multivariate normal observation (or between a pair of observations).

 $information(x_i) = -\log(p(x_i))$ 

$$p_{normal}(x_i) \propto e^{-\frac{1}{2}(x_i - \bar{x})\Omega^{-1}(x_i - \bar{x})'}$$

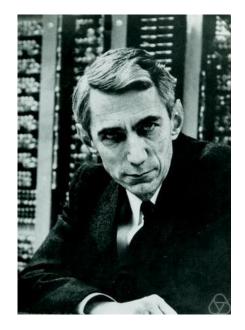
information( $x_i$ )  $\propto (x_i - \bar{x})\Omega^{-1}(x_i - \bar{x})'$ 



# Why relevance is not arbitrary (Part 1: information theory)

- Information theory tells us that the information in an observation is the negative logarithm of its likelihood.
- It also tells us that the normal distribution has **maximum entropy** for a given variance.
- In addition, the Central Limit Theorem motivates the normal distribution as a common limiting case of many random processes.
- The **Mahalanobis distance** measures the information in a multivariate normal observation (or between a pair of observations).









## Why relevance is not arbitrary (Part 2: linear regression equivalence)

- Relevance values sum to zero and tilt around equal weights.
- Recall that we form a prediction as a weighted average of past outcomes.
- The prediction matches linear regression (OLS) if we include every observation in our data sample.

$$w_{it,lin} = \frac{1}{N} + \frac{1}{N-1}r_{it}$$
$$\hat{y}_t = \sum_{i=1}^{N} w_{it}y_i$$
$$\hat{y}_{t,lin} = \bar{y} + \sum_{i=1}^{N} r_{it}(y_i - \bar{y})$$
$$\hat{y}_{t,lin} = \alpha + x_t\beta$$
Equal



# Why relevance is not arbitrary (Part 2: linear regression equivalence)

- Relevance values sum to zero and tilt around equal weights.
- Recall that we form a prediction as a weighted average of past outcomes.
- The prediction matches linear regression (OLS) if we include every observation in our data sample.



Source: Public domain image of portrait by Christian Albrecht Jensen.



### **Partial sample regression**

This equivalence with linear regression reveals an intriguing insight:

Linear regression places as much importance on non-relevant observations as on relevant ones.

It just changes the sign of their effect.



### **Partial sample regression**

- **Partial sample regression** forms a prediction from a subset of the most relevant observations.
- We may write the prediction equivalently in terms of weights that sum to one.
- Predictions are no longer linear with respect to prediction circumstances (x<sub>t</sub>). Censoring introduces nonlinear conditionality.

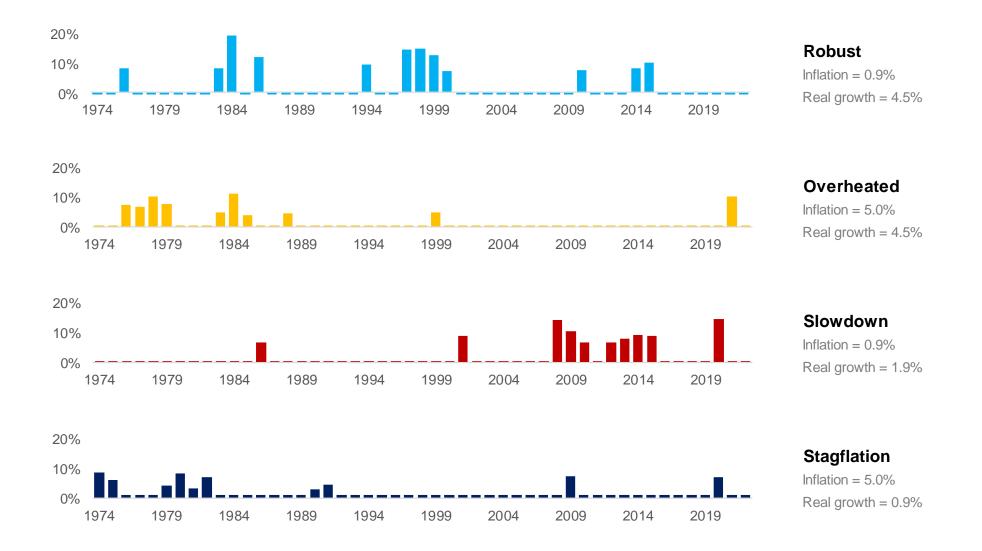
$$\hat{y}_{t,psr} = \bar{y} + \frac{\lambda^2}{n-1} \sum_{i} \delta(r_{it}) r_{it} (y_i - \bar{y})$$

$$w_{it,psr} = \frac{1}{N} + \frac{\lambda^2}{n-1} (\delta(r_{it})r_{it} - \varphi \bar{r}_{sub})$$

**DEFINITIONS:** 
$$\delta(r_{it}) = 1 \text{ if } r_{it} > r^*, \text{ else } 0$$
  
 $n = \sum_i \delta(r_{it})$   $\bar{r}_{sub} = \frac{1}{n} \sum_i \delta(r_{it}) r_{it}$   
 $\varphi = \frac{n}{N}$   $\lambda^2 = \frac{\sigma_{r,full}^2}{\sigma_{r,partial}^2} = \frac{\frac{1}{N-1} \sum_i r_{it}^2}{\frac{1}{n-1} \sum_i \delta(r_{it}) r_{it}^2}$ 



### Economic example: Partial sample prediction weights (inflation & growth)



Notes: Weights are based partial sample regression using the 25% most relevant observations between 1974 and 2022. Inflation and real growth data are obtained from the FRED data website. Source: "Portfolio Construction When Regimes Are Ambiguous" by Kritzman, Kulasekaran and Turkington.

### 5940080.1.1.GBL.



20

## **Binary weights versus partial sample weights**

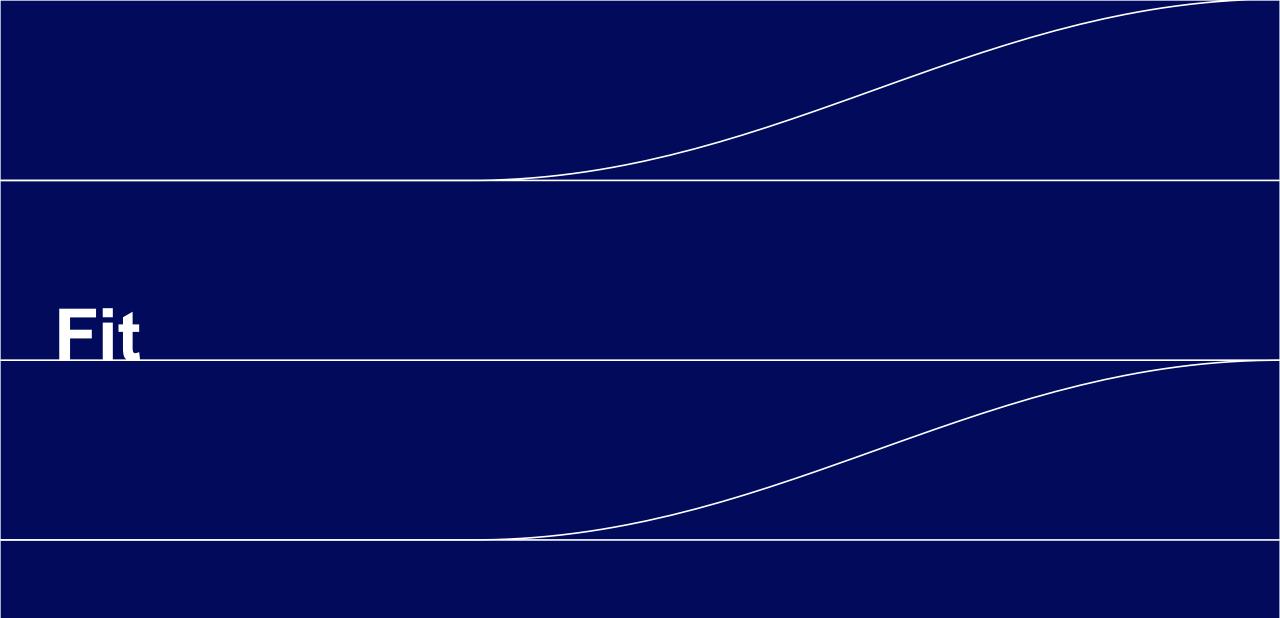
### **Binary weights**

- It is typical to define regimes as binary occurrences based on a single indicator and to estimate regime outcomes by equally weighting their values across the regime periods.
- This binary approach assumes that all regime **periods are equally useful** in predicting outcomes.
- It precludes using more than one indicator in a principled way, and it risks producing **empty regimes**.

### Partial sample weights

- Partial sample prediction instead weights past outcomes by the relevance of each regime period.
- Relevance reflects the extent to which a period aligns with a regime's prototypical circumstances.
- It allows a period to be connected to more than one regime.
- And it enables us to consider regimes that have **never occurred** historically.





5940080.1.1.GBL.



- Fit reveals how much confidence we should attach to a **specific prediction task**, separately from the overall confidence we have in the associated prediction routine.
- Fit is equal to the squared correlation between observation weights and outcomes.

 $fit_t = \rho(w_t, y)^2$ 

• Note that this calculation is only possible because we know each observation's contribution to the prediction.



### Why relevance is not arbitrary (Part 3: R-squared equivalence)

• For a full-sample linear regression, the informativeness-weighted average fit across all prediction tasks equals the classical *R*<sup>2</sup> statistic.

$$R^{2} = \frac{1}{T-1} \sum_{t} info(x_{t}) fit_{t}$$

• The fit of each prediction can vary dramatically, which underscores the limitation of *R*<sup>2</sup> as a measure of confidence for an individual prediction.



# Codependence



5940080.1.1.GBL.

## Using fit to optimize prediction-specific calibration

- For partial sample regression, rather than choose the censoring threshold  $r^*$  arbitrarily, we compute fit for a range of possible values and select the value that maximizes fit.
- Fit inherently balances small sample noise with subsample fit.

$$fit_{t,psr}(w) = \left(\frac{\varphi\lambda^2}{1-\varphi c^2}\right) \frac{N}{N-1} \frac{n-1}{n} fit_t(\delta(r_t)r_t, y)$$

$$\bigwedge Noise Diagonal correction Gubsample fit$$

DEFINITIONS:

26

$$c^{2} = \frac{\bar{r}_{sub}^{2}}{\sigma_{r,partial}^{2}} \frac{n}{n-1}$$

### Codependence

- We may also use fit to select **prediction-specific** subsets of X variables.
- We maximize fit as a **joint function** of the choice of observations and the choice of variables.
- We refer to the prediction routine based on this joint maximization as **CKT regression**.
- **CKT regression** recognizes that the usefulness of observations and variables is **not static nor mutually independent**; it depends on the prediction task's unique circumstances.
  - Lasso regression selects variables but assumes a variable is useful always or never.
  - Kernel regression selects observations but assumes a static calibration.
  - Moreover, it is suboptimal to apply these selection techniques independently.



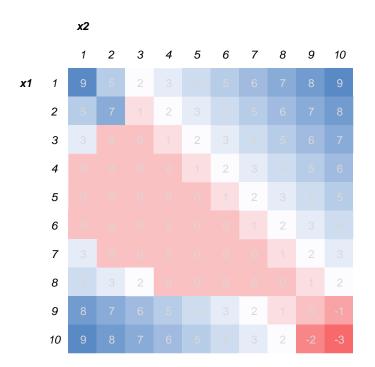
# **Empirical Illustrations**



5940080.1.1.GBL.

### **Toy example: Two predictive variables**

### Observed (contrived) data



# CKT predictions

5

6

7

8

6.2 6.6 7.5 8.1

9 10

3

1

2

3

4

5

6

7

8

9

10

4

# Linear regression predictions

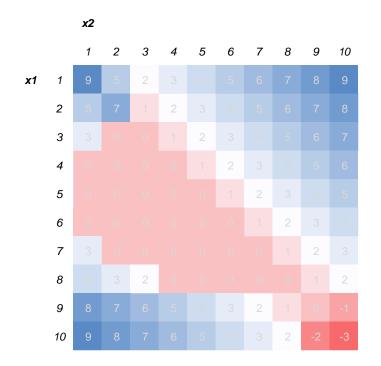
	1	2	3	4	5	6	7	8	9	10
1				3.8				4.0		4.2
2										3.9
3										
4	2.8									
5			2.7	2.8						
6							2.8	2.8		
7										2.7
8										
9										
10										2.0

Correlation with actual: 73%

### Correlation with actual: 26%

### **Toy example: Prediction-specific confidence**

### Observed (contrived) data



# Prediction-specific fits (CKT)

	1	2	3	4	5	6	7	8	9	10
1	0.2	0.2	0.2	0.2	0.2	0.3				0.4
2	0.2	0.2	0.2	0.2	0.2					
3	0.2	0.2	0.2	0.2	0.2					0.3
4	0.2	0.2	0.2	0.2	0.2				0.2	0.2
5	0.2	0.2	0.2	0.2	0.2		0.2	0.2	0.2	0.2
6					0.2					0.1
7				0.2	0.2					0.1
8			0.2	0.2	0.2					0.1
9	0.2	0.2	0.2	0.2						0.1
10	0.2	0.2	0.2	0.2						0.1

### Toy example: Codependent observation and variable selection

2

1

2

3

4

5

6

7

8

9

10

3

4

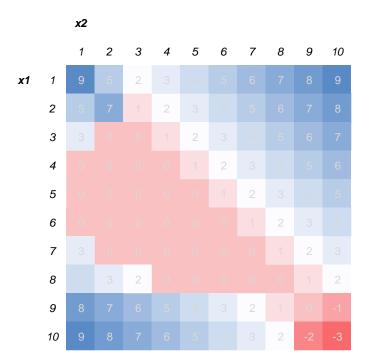
0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2

0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.3

0.2 0.2 0.2 0.2 0.2 0.2 0.3

0.2 0.2 0.2 0.2 0.2 0.3

### Observed (contrived) data



# Prediction-specific $r^*$

5

0.2 0.2 0.2 0.2 0.2 0.3 0.2 0.2 0.2 0.2

6

7

9

0.2 0.2 0.2

10

### Prediction-specific inclusion of x2

	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										



## **Regime simulation: Model assumptions**

### Assumptions

- Assume a Regime 1 with 80% persistence, and a Regime 2 with 60% persistence.
- In Regime 1, A and B are tightly distributed around a positive average and causally determine the outcome Y, while C and D contribute noise.
- In **Regime 2**, C and D are tightly distributed around a positive average and causally determine the outcome Y, while A and B contribute noise.
- A and B are 50% correlated, but independent of C and D.
   C and D are 50% correlated, but independent of A and B.
- The formula for Y is deterministic for a given regime, but the current regime is unknown and must be inferred from the predictive variables.

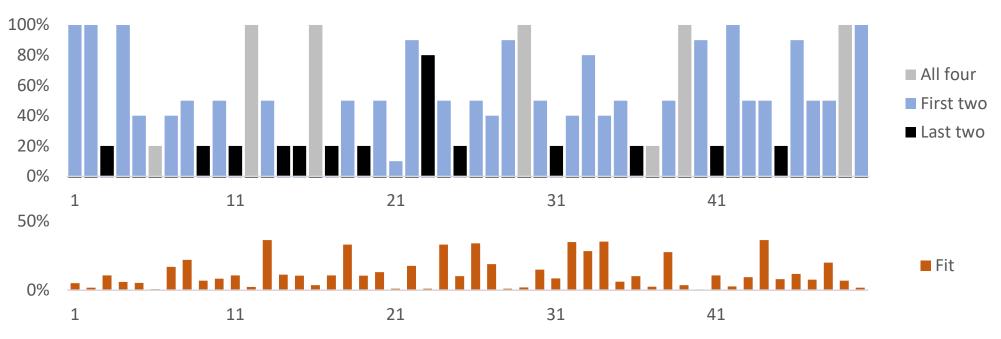
### Procedure

- We simulate 500 observations as a training sample, and an additional 50 for testing (taking every tenth observation of a new simulated sample).
- For each prediction task from the testing sample, we compute the fit for relevance-based predictions based on different collections of variables (all four, the first two, and the last two) and different thresholds for observation relevance (increments of 0.1 from 0 to 0.9).
- We select the specification with the greatest fit and evaluate this prediction against the actual testing outcome.



### **Regime simulation: Results**

- Out-of-sample outcomes are 89% correlated to CKT predictions (versus 54% for linear regression).
- Absolute errors are -43% correlated to relevance-based fits (versus -15% for linear regression).



### Fraction of observations included (top) and fit (bottom)

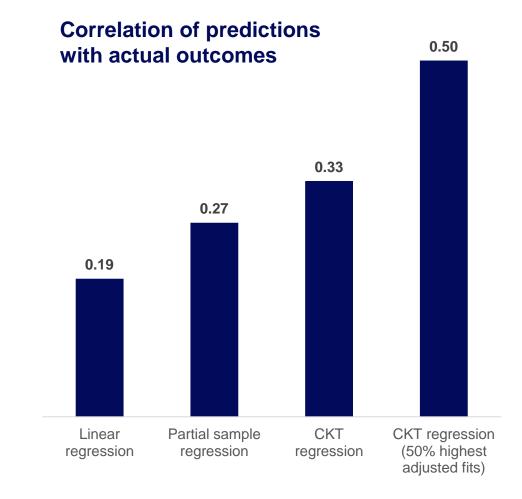
For illustration only. Source: State Street Global Markets.

5940080.1.1.GBL.



## **Empirical application: Predicting NBA draft prospect performance**

- Predict BPM (average box score plus/minus) for the 2018-2019 season for 39 rookie draft picks.
- Training sample: 249 previously drafted players from 2011 to 2017.
- Predictive variables: physical attributes (2), performance in final college season (10), college team success (2), draft team success (2).
- CKT regression, allowing relevance/similarity censoring, variable selection, and adjusted fit to reflect asymmetry and number of predictive variables.

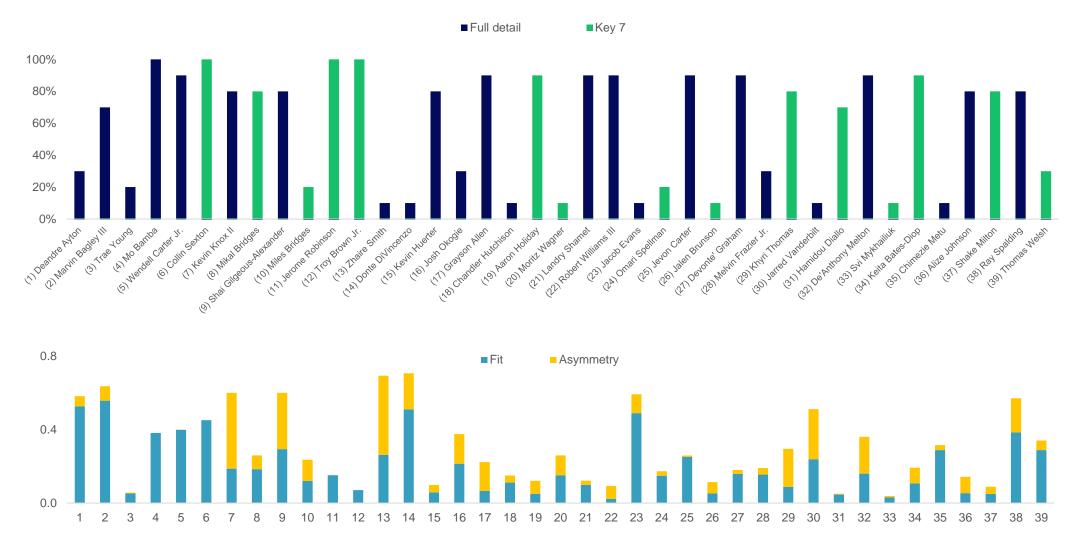


Source: "How to Predict the Future Performance of NBA Draft Prospects" by Czasonis, Kritzman, Kulasekaran and Turkington (Working paper, 2023).



34

### **Empirical application: Predicting NBA draft prospect performance**



Source: "How to Predict the Future Performance of NBA Draft Prospects" by Czasonis, Kritzman, Kulasekaran and Turkington (Working paper, 2023).

#### 5940080.1.1.GBL.



## **Comparative summary**

	Linear regression (LR)	Machine learning (ML)	Relevance-based prediction (RBP)
Transparency	Despite its simplicity, LR is silent about the impact of each observation and the quality of individual predictions.	The most powerful ML algorithms are notoriously opaque.	RBP shows how observations contribute to a prediction and quantifies confidence for each prediction task.
Adaptability	LR works well only for static linear relationships.	Despite their capacity to model complexity, ML models do not adapt their information use to new prediction circumstances.	RBP identifies the optimal sample of observations and variables simultaneously for each prediction task.
Non- arbitrariness	LR has a strong theoretical justification.	ML is guided by empirical efficacy rather than a core set of theoretical principles.	RBP is theoretically justified by both the Central Limit Theorem and information theory and is mathematically unified with linear regression.

### **Disclaimers and Important Risk Information**

#### **Disclaimers and Important Risk Information [2023.01]**

This communication is provided only to professional clients or eligible counterparties or their equivalent by State Street Bank and Trust Company or, where applicable and permissible, its bank and non-bank affiliates ("State Street"). State Street Bank and Trust Company is authorized and regulated by the Federal Reserve Board, registered with the Commodity Futures Trading Commission as a Swap Dealer, and is a member of the National Futures Association. State Street Bank International GmbH ("ECB"), the German Federal Financial Supervisory Authority ("BaFin") and the Deutsche Bundesbank. Details about the extent of SSBI's regulation by the ECB, the BaFin and Deutsche Bundesbank are available from offshore. Information provided is of a general nature only and has not been reviewed by any regulatory authority.

This communication is intended for general marketing purposes, and the information contained herein has not been prepared in accordance with legal requirements designed to promote the independence of investment research. It is for clients to determine whether they are permitted to receive research of any nature. Market commentary provided by trading desks is not investment research. This communication is not intended to suggest or recommend any transaction, investment, or investment strategy, does not constitute investment research, nor does it purport to be comprehensive or intended to replace the exercise of an investor's own careful independent review and judgment regarding any investment decision.

This communication is not intended for retail clients, nor for distribution to, and may not be relied upon by, any person or entity in any jurisdiction or country where such distribution or use would be contrary to applicable law or regulation. This communication or any portion hereof may not be reprinted, sold or redistributed without the prior written consent of State Street. This communication and the information herein does not constitute investment, legal, or tax advice and is not a solicitation to buy or sell securities or any financial instrument nor is it intended to constitute a binding contractual arrangement or commitment by State Street of any kind. The information provided does not take into account any particular investment objectives, strategies, investment horizon or tax status.

The views expressed herein are the views of State Street as of the date specified and are subject to change, without notice, based on market and other conditions. The information provided herein has been obtained from sources believed to be reliable at the time of publication, nonetheless, we make no representations or assurances that the information is complete or accurate, and you should not place any reliance on said information. State Street hereby disclaims any warranty and all liability, whether arising in contract, tort or otherwise, for any losses, liabilities, damages, expenses or costs, either direct, indirect, consequential, special, or punitive, arising from or in connection with any use of this document and/or the information herein.

State Street may from time to time, as principal or agent, for its own account or for those of its clients, have positions in and/or actively trade in financial instruments or other products identical to or economically related to those discussed in this communication. State Street may have a commercial relationship with issuers of financial instruments or other products discussed in this communication.

This communication may contain information deemed to be forward-looking statements. These statements are based on assumptions, analyses and expectations of State Street in light of its experience and perception of historical trends, current conditions, expected future developments and other factors it believes appropriate under the circumstances. All information is subject to change without notice.

Participating in trading any financial instrument, including but not limited to foreign exchange, equities, futures, fixed income or derivative instruments, or investments in non-liquid or emerging markets, or digital assets, or participating in securities lending, repurchase transactions or other collateral services present risks, which may include but are not limited to counterparty, collateral, investment loss, tax, and accounting risks. Where applicable, returns may increase or decrease as a result of currency fluctuations. Derivatives may be more volatile than the underlying instruments. Certain foreign exchange business, including spot and certain forward transactions, may not be regulated in all jurisdictions. Past performance is no guarantee of future results.

Please contact your State Street representative for further information.

To learn how State Street looks after your personal data, visit: https://www.statestreet.com/utility/privacy-notice.html.

© 2023 State Street Corporation – All Rights Reserved

#### Global Markets Research Disclaimer Supplement [2023.01]

Australia: This communication is provided to wholesale clients by State Street Bank and Trust Company (Australian Business Number 70 062 819 630, Australian Financial Services License 239679).

Brazil: The products in this marketing material have not been and will not be registered with the Comissão de Valores Mobiliários - the Brazilian Securities and Exchange Commission ("CVM"), and any offer of such products is not directed to the general public within the Federative Republic of Brazil ("Brazil"). The information contained in this marketing material is not provided for the purpose of publicly soliciting investments from investors residing in Brazil and no information in this marketing material should be construed as a public offering or unauthorized distribution of the products within Brazil, pursuant to applicable Brazilian law and regulations.

Israel: State Street Bank and Trust Company is not licensed under Israel's Regulation of Investment Advice, Investment Marketing and Portfolio Management Law, 1995. This communication may only be distributed to or used by investors in Israel which are "eligible clients" as listed in the First Schedule to Israel's Regulation of Investment Advice, Investment Marketing and Portfolio Management Law 1995.

Japan: This communication is made available in Japan by State Street Bank and Trust Company, Tokyo Branch, which is regulated by the Financial Services Agency of Japan and is licensed under Article 47 of the Banking Act.

Oman: State Street Bank and Trust Company is not a bank or financial services provider registered to undertake business in Oman and is not regulated by the Central Bank of Oman or the Capital Market Authority.

Qatar: The information in this communication has not been reviewed or approved by the Qatar Central Bank, the Qatar Financial Markets Authority or the Qatar Financial Centre Regulatory Authority, or any other relevant Qatari regulatory body.

Singapore: This communication is made available in Singapore by State Street Bank and Trust Company, Singapore Branch ("SSBTS"), which holds a wholesale bank license by the Monetary Authority of Singapore. In Singapore, this communication is only distributed to accredited, institutional investors as defined in the Singapore Financial Advisers Act 2001 ("FAA") and its regulations. Note that SSBTS is exempt from Sections 27 and 36 of the FAA. When this communication is distributed to overseas investors as defined in the FAA, note that SSBTS is exempt from Sections 26, 27, 29 and 36 of the FAA. This advertisement has not been reviewed by the Monetary Authority of Singapore.

South Africa: State Street Bank and Trust Company is authorized in South Africa under the Financial Advisory and Intermediary Services Act, 2002 as a Category I Financial Services Provider; FSP No. 42671.

United Arab Emirates: The information contained within this communication is not intended to lead to the conclusion of any contract of whatsoever nature within the territory of the United Arab Emirates.

United Kingdom: State Street Bank and Trust Company is authorised and regulated by the Federal Reserve Board of the United States, authorised by the Prudential Regulation Authority ("PRA") and subject to regulation by the Financial Conduct Authority and limited regulation by the PRA. Details about the extent of our regulation by the PRA are available from us on request.

State Street Bank International GmbH is authorised and regulated by the European Central Bank and the BaFin, deemed authorised by the Prudential Regulation Authority, and subject to regulation by the Financial Conduct Authority and limited regulation by the Prudential Regulation Authority. Details of the Temporary Permissions Regime, which allows EEA-based firms to operate in the UK for a limited period while seeking full authorisation, are available on the Financial Conduct Authority's website.

37

STATE STREET