Getting to the Core: Inflation Risks Within and Across Asset Classes[*]

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Abstract

Do “real” assets protect against inflation? Core inflation betas of stocks are negative while energy betas are positive; currencies, commodities, and real estate also mostly hedge against energy inflation but not core. These hedging properties are reflected in the prices of inflation risks: only core inflation carries a negative risk premium, and its magnitude is consistent both within and across asset classes, uniquely among macroeconomic risk factors. The relative contribution of core and energy changes over time, helping explain the time-varying correlation between stock and bond returns. A two-sector New Keynesian model qualitatively accounts for these facts.

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1 Introduction

Inflation is a key macroeconomic factor and a fundamental source of risk driving asset returns. Conventional wisdom holds that fixed-income securities incur losses in the face of inflation, but stocks, foreign currencies, commodities, and real estate maintain their values in real terms. Stocks are claims on real physical assets. Foreign currencies should appreciate when domestic price levels rise. Commodity and real estate prices are important components of total inflation in their own right. If investors fear inflation, they might accept lower average returns on assets that provide some protection against inflation risk (Chen, Roll, and Ross, 1986). Yet empirical evidence of such a risk premium has been elusive, as have the inflation-hedging properties of supposedly "real" assets, notably stocks (see, e.g., Fama and Schwert (1977), Bekaert and Wang (2010), Katz, Lustig, and Nielsen (2017)).

We demonstrate that decomposing inflation into core and non-core components (particularly energy) is important as it sheds new light on the nature of inflation risks. First, core and energy inflation series have sharply different statistical and economic properties. Second, inflation-hedging properties of conventional "real assets," such as stocks, currencies, commodity futures, and real estate investment trusts (REITs), are largely confined to energy inflation. These assets provide almost no protection against the core inflation risk. Third, core inflation carries a significantly negative price of risk, while the risk price associated with energy inflation is positive but statistically indistinguishable from zero. These estimates of risk prices provide a new perspective on the risk-return trade-off across a wide range of asset classes as well as their comovement.

Historically, core inflation has been much more stable and persistent than energy inflation. Core and energy inflation series have a very low correlation, despite both being highly correlated with headline inflation. Economically, core goods' prices have a substantially higher degree of rigidity than energy prices, and are potentially driven by different supply and demand shocks. These distinctions, largely glossed over in the literature, can potentially lead to very different ways in which inflation risks manifest themselves in asset prices.

We begin by estimating a VAR that allows us to separate innovations to the headline inflation and its components. Armed with this decomposition, we revisit the inflation-hedging properties of different assets. We examine 8 major asset classes: U.S. stocks, Treasury notes/bonds, agency bonds, corporate bonds, currencies, commodity futures, REITs, and international stocks. The broad coverage of assets is informative since investors often consider multi-asset-class allocations when it comes to managing macroeconomic risk, and inflation in particular. Our estimates of headline inflation betas confirm, to some extent, the conventional view on inflation hedging. Fixed income securities have negative headline inflation betas, while currencies, commodities, and REITs have positive betas. Stocks' headline betas are mostly negative but often statistically insignificant. However, after decomposing headline inflation into core and energy, we find that assets' exposures to the two components are sharply different. All stock and REITs portfolios have consistently negative core betas and positive energy betas, which means stocks and REITs hedge against energy inflation and are hurt by core inflation. Treasuries and agency bonds are negatively exposed to both core and energy inflation shocks, and corporate bonds have negative core betas and insignificant energy betas. The exposures of currencies and commodity futures to energy inflation are positive, and those to core inflation are negative but insignificant. Therefore, the conventional view mixes the two distinct components of inflation, core and energy, in a way that potentially obscures their effects on asset prices. For example, stocks' often insignificant headline betas are largely an artefact of their energy betas, which vary over time but are strongly positive in the recent decades, obscuring the robustly negative betas with respect to core inflation. Currencies, commodities, and REITs, often considered as inflation-hedging assets, also only hedge against the energy inflation but not the core.

Next, we study the cost of hedging against inflation shocks. In other words, we ask, how much return are investors willing to give up to hold assets that do well when inflation is unexpectedly high? The cost of inflation hedging, or, equivalently, the price of inflation risks, reflects investors' attitude toward inflation. The higher the cost, the more investors are averse to inflation. To answer this question, we conduct cross-sectional asset pricing tests using both the 8 average portfolios in each asset class and a larger cross section of 38 test portfolios. The price of headline inflation risk is around zero and insignificant, which seems to indicate that hedging against inflation is free, consistent with the elusive inflation risk premium in the previous literature. However, when we differentiate core from energy inflation, a different picture emerges. Core inflation carries a significantly negative price of risk, and the price of energy inflation risk is positive but indistinguishable from zero. In other words, hedging against core inflation is costly, while hedging against energy inflation is essentially "free" or even rewarded. The empirical results on the price of core and energy inflation risks are robust to controlling for various measures of real consumption risk and other macroeconomic factors.

We further take advantage of the multi-asset-class setting to estimate the price of inflation risks within each asset class. Strikingly, the magnitude of the core inflation risk price is rather consistent across asset classes. Average returns of assets line up well with core inflation betas both within and across asset classes, but are essentially unrelated to betas with energy or headline inflation. Therefore, different asset classes imply a largely consistent cost of hedging against core inflation. We construct the factor mimicking portfolios for headline, core, and energy inflation using portfolios from each asset class. Only the average returns of core inflation mimicking portfolios behave similarly, while the average returns of headline and energy inflation mimicking portfolios are unstable and switch signs for different asset classes.

In addition to the unconditional inflation risk exposures, we examine how these exposures - and their prices - vary over time. Splitting the sample allows us to investigate the time-varying inflation risk exposures. Stocks, currencies, commodity futures, REITs, and international stocks have energy betas that are significantly larger in the post-1999 subsample than before, while their core betas did not experience significant changes. Moreover, core inflation becomes less volatile after the 1980s while energy inflation fluctuates more wildly in the 2000s. The growing dominance of energy in total inflation (relative to core) and its positive correlation with stock returns serves as a potential new explanation for why the correlation between bond and stock returns switches from positive to negative in the recent subsample (Song, 2017; Campbell, Pflueger, and Viceira, 2020). Before 1999, core inflation is relatively more important. Both stocks and bonds have negative core betas, so their correlation is positive. After 1999, energy inflation becomes dominant, and stocks and bonds have different energy betas in sign. As a result, the correlation between stock and bond returns turns from positive to negative. Stocks' energy betas are significantly smaller in the early sample, consistent with the view that energy supply mainly drives energy inflation pre-1999 while demand dominates after 2000 (Baumeister and Hamilton, 2018; Kilian, 2009; Ready, 2018).

Why do investors fear core inflation? In a seminal paper, Fama (1981) proposed that low inflation is a proxy for high growth of real activity, leading to the negative inflation betas of stocks. Our results show that core inflation is an important risk by itself, and, while it leads output, dividend, and consumption growth in the manner that Fama suggested, it is not driven out by standard measures of real activity. This result echoes the findings in the New Keynesian DSGE literature that argues inflation "dances to its own tune." (Smets and Wouters, 2007) The negative correlation between core inflation and subsequent real activity helps explain its impact on stocks, in particular. A decomposition of stock returns into cash flow (CF) and discount rate (DR) news shows that both components contribute to the negative core beta of stock portfolios.

Do markets react strongly to core inflation news because they fear an aggressively hawkish response of monetary policy? Utilizing high-frequency data over short windows around inflation announcements, we find that the policy rate expectations of market participants, such as those embedded in Fed Funds Futures, do respond to core inflation and thus depress stock returns on positive core inflation surprises. However, the effect of core inflation on stock returns is still pronounced even after controlling for the change in monetary policy expectations, which indicates that stocks' negative core inflation beta might not be driven entirely by the response of monetary policy.

Why is core inflation risk different from energy? We explore a potential mechanism empirically by looking at inflation of goods whose prices are sticky and flexible. We find that sticky and flexible inflation resemble the properties of core and energy inflation, respectively, in both their risk exposures and risk premia. In order to interpret our empirical findings, we develop a two-sector New Keynesian model that rationalizes the stylized facts listed above: (i) stocks are negatively exposed to core inflation; (ii) Treasuries (and nominal bonds more generally) are negatively exposed to both core and energy inflation; (iii) currencies and commodity futures are positively exposed to energy inflation; (iv) core inflation carries a negative price of risk while the price of energy inflation risk is positive but potentially difficult to distinguish from zero due to the countervailing effects of energy demand and supply shocks.

Our model is qualitative and includes only the minimum set of ingredients necessary to account for our empirical findings. The model features a small-scale New Keynesian economy with an energy sector. Households consume both core and energy goods. There is a continuum of varieties of core goods, and each variety is produced by a monopolistic firm that chooses to set the nominal price of the good. Firms face price stickiness, i.e., only a fraction of firms can adjust their prices freely. The desired markup fluctuates exogenously and is the main driver of core inflation. The fluctuations in desired markups capture the variation of inflation that is independent of other real macroeconomic and policy shocks. Energy goods are subject to energy supply and demand shocks and, importantly, face no price rigidity.

In this economy, when a positive markup shock increases the cost of production, core inflation rises and core output drops, and thus the marginal utility of consumption increases. Therefore, core inflation carries a negative price of risk. Stocks, which are claims to the core output, are negatively exposed to core inflation. Nominal returns of Treasuries have negative betas with core and energy inflation since the values of Treasuries decrease with inflation and its expectations. Furthermore, energy supply and demand shocks have opposite effects on energy inflation, but both shocks are expansionary to core production when core and energy goods are complementary. Therefore, a positive energy supply and demand shock implies a higher consumption demand of the core good. A high energy price combines the effect of a negative supply and a positive demand, which have opposite effects on core output, core inflation, and thus the stochastic discount factor in their signs. As a result, energy inflation carries a price of risk that is hard to identify due to the two offsetting forces. When the demand is the major driver of energy inflation, the model implies a positive price of energy inflation, positive energy betas for stocks and currencies and negative betas for Treasuries, which are consistent with the data.

Related Literature A large body of literature studies the inflation-hedging properties of financial assets (Fama and Schwert, 1977; Boudoukh and Richardson, 1993; Bekaert and Wang, 2010). Stocks have a negative correlation with unexpected inflation, thus being poor inflation hedges. Fama (1981) argues that the negative correlation is caused by real activity that is correlated positively with stock returns and negatively with inflation. Katz, Lustig, and Nielsen (2017) propose that stock investors respond slowly to inflation. Nominal bond returns negatively covary with inflation. Commodity futures returns are positively correlated with inflation (Gorton and Rouwenhorst, 2006), but the correlation with core inflation is weak (Kogan and Konstantinovsky, 2008).

The cost of inflation hedging, a.k.a. the inflation risk premium, has been of great interest for the literature exploring the macroeconomic sources of risks embedded in asset prices. In the stock market, Chen, Roll, and Ross (1986) document a marginally negative price of inflation risk. Boons, Duarte, de Roon, and Szymanowska (2019) show that inflation risk is priced in the inflation-beta-sorted stock portfolios but its sign changes from negative to positive after 2000, consistent with the role of energy inflation that we document. Inflation risk is a common feature in term structure models that distinguish between real and nominal bonds (Ang, Bekaert, and Wei, 2008). While most of this work has focused on the headline inflation, one important exception is Ajello, Benzoni, and Chyruk (2019) who incorporate both core and "crust" inflation components into an affine term structure model. Fleckenstein, Longstaff, and Lustig (2017) find an important role for deflation risk embedded in the prices of inflation derivatives. Kang and Pflueger (2015) demonstrate credit spreads rise with inflation volatility as inflation erodes the real value of corporate liabilities and, conversely, low inflation increases credit risk; and Augustin, Cong, Corhay, and Weber (2021) explore the relation between price rigidity and credit risks in the cross section. In currencies, Hollifield and Yaron (2003) find little evidence of an inflation risk premium. Clarida and Waldman (2008) show that the relation between inflation and exchange rate reflects the conduct of monetary policy, leading to a negative reaction of foreign exchange rates to domestic inflation if domestic monetary policy is sufficiently proactive; Stavrakeva and Tang (2019) uncover the information channel of monetary policy that drives inflation and exchange rates. Andrews, Colacito, Croce, and Gavazzoni (2020) show that inflation risks in different countries explain the term structure of carry trade returns and its time variation. In contrast to much of this work, we utilize a large cross section of multiple asset classes, including portfolios of stocks, government, agency, and corporate bonds, currency portfolios (Lustig, Roussanov, and Verdelhan, 2011, 2014; Menkhoff, Sarno, Schmeling, and Schrimpf, 2017; Verdelhan, 2018) and commodity futures portfolios (Bakshi, Gao, and Rossi, 2019) and find largely consistent magnitudes of inflation risk premia across them.

Among theoretical studies of inflation risks and asset prices, several equilibrium models with an endowment economy can quantitatively match inflation, term structure and stock returns (Buraschi and Jiltsov, 2005; Wachter, 2006; Piazzesi and Schneider, 2006; Bansal and Shaliastovich, 2012). In Bansal and Shaliastovich (2012), the inflation premium is due to the negative effect of inflation on future long-run real growth. Eraker, Shaliastovich, and Wang (2016) further find the inflation-growth effect is more pronounced in the durable goods sector, and durable stocks are more exposed to inflation risks. Kung (2015) builds a New Keynesian model with an endogenous interaction between inflation and real growth in order to account for several stock and bond price puzzles. Campbell, Pflueger, and Viceira (2020) estimate a New Keynesian model and show that time-varying stock-bond correlation is driven by monetary policy regimes. Weber (2015) shows that firms facing stronger price rigidity earn a premium. Gomes, Jermann, and Schmid (2016) explore the real consequences and monetary policy implications of sticky leverage, while Bhamra, Dorion, Jeanneret, and Weber (2018) study the impact of inflation on default risk and equity valuation in the presence of both sticky leverage and price rigidity. Song (2017) estimates a regime-switching model and variations in the cyclical properties of inflation and its premium. To explain the new facts that we document, we propose a New Keynesian model with production and an energy sector and study it analytically. Our model highlights the role of the markup shock as the major source of priced inflation risk. This is consistent with Smets and Wouters (2007) that emphasize that inflation is mostly driven by markup shocks in a quantitative New Keynesian DSGE model.

This paper also contributes to studying the interaction between commodities and other asset prices (Ready, Roussanov, and Ward, 2017a,b; Ready, 2017). Barro and Misra (2016) study gold returns in the historical data and in a rare disaster model and find that gold cannot hedge real macroeconomic risks. Similar to their finding, we show that gold is not a good hedge for core inflation either. Commodities are not only an asset class but also a source of macroeconomic risk in their own right. We explicitly model and analyze the role of energy commodities in driving inflation risk. There is an ongoing debate in the literature regarding the relative importance of different types of shocks (e.g. supply vs. demand) in driving the prices of key energy commodities such as crude oil (Kilian, 2009; Baumeister and Hamilton, 2018; Ready, 2018). We contribute to this literature by bringing in asset prices, which move differently in response to shocks to energy demand and supply, as an additional source of identifying variation.

A series of studies examine the changing signs of the stock-bond correlation in the late 1990s (Campbell, Sunderam, and Viceira, 2009; Song, 2017; Campbell, Pflueger, and Viceira, 2020; Cieslak and Pang, 2021). The breakdown of inflation in our analysis into core and energy components, with their changing importance over time, provides a complementary explanation of this important phenomenon.

2 Empirical Analysis

2.1 Data and Descriptive Statistics

2.1.1 Inflation

We use the consumer price index (CPI) and its components from the U.S. Bureau of Labor Statistics as inflation measures.[1] The CPI has three components: core (CPI less food and energy), food, and energy. The expenditure categories in core include shelter, household furnishings and operations, apparel, transportation, medical care, recreation, education and communication, alcoholic beverages, and others goods and services (tobacco, personal care, etc). The sample is at the quarterly frequency from 1963Q2 to 2019Q4.

The Panel A of Table 1 reports the summary statistics of inflation. The headline inflation (CPI) and its three components have similar average of about 4 percent per annum over the sample. Core, food, and energy differ greatly in their volatility and persistence. Core inflation has low volatility and high persistence, with a standard deviation of 2.66 percent per annum and an autocorrelation of 0.79. In contrast, energy inflation is much more volatile with a standard deviation of 19.52 percent per annum and exhibits little persistence. The food inflation stands between core and energy inflation in both volatility and persistence. The large difference in their persistence can be attributed to the different degrees of price rigidity in those goods. Core goods and services, such as apparel, shelter, medical care, feature stronger price rigidity, while energy prices are flexible.

Panel B of Table 1 reports the relative weights of the three components in the headline inflation. These weights are obtained by regressing the headline inflation onto the three components. Core inflation accounts for 71 percent of headline inflation, food accounts for 20 percent, and energy accounts for the least, only 9 percent of headline inflation. Although core inflation accounts for the largest portion, energy inflation is much more volatile and substantially drives headline inflation fluctuations as well.

In Panel C of Table 1, we examine the correlation structure of headline inflation and the three components. All three components are fairly correlated with headline inflation (core 0.80, food 0.60 and energy 0.69). However, the correlations across the three components are much lower. Energy

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[1] An alternative measure of inflation is the price level index for personal consumption expenditure (PCE). In Appendix C.4, we report similar results using PCE instead of CPI.

inflation is correlated with neither core (0.20) nor food inflation (0.17), while food and core inflation has a moderate correlation of 0.44.

To summarize, the three components of headline inflation have distinct volatility and persistence, and they are not correlated with each other, especially the core and energy components. Because energy inflation exhibits a stark contrast with the core inflation, we focus on the core and the energy part of the noncore inflation, and leave out the food part for parsimony.

2.1.2 Asset Returns

We use test portfolios from a wide and standard asset classes: stocks, Treasuries, agency bonds, corporate bonds, currencies, commodity futures, REITs, and international stocks. We first consider an average portfolio in each asset class. An average portfolio for stock, agency bond, commodity future, REITs, and international stock is constructed using the respective market index. The average Treasury and corporate bond portfolio returns are the average of the cross-sectional portfolios below. The average currency portfolio is the equal-weighted average of the six interest rate sorted carry portfolios.

We examine a wider cross section by including a set of portfolios in each asset class. These assets include 5 industry stock portfolios (consumer, manufacturing, high tech, health, and others), 7 fixed-term Treasury portfolios, 4 maturity sorted agency bond portfolios, 4 maturity sorted corporate bond portfolios, 6 interest rate sorted currency carry portfolios (Lustig, Roussanov, and Verdelhan, 2011) and the dollar carry portfolio (Lustig, Roussanov, and Verdelhan, 2014), 5 commodity future portfolios of major categories (livestock, industrial metal, precious metal, energy, and agriculture), 3 REITs portfolios (equity, mortgage, and hybrid), and 3 regional international stocks (North America, Europe, Far East). These data are obtained from different sources: stock returns are from Ken French's website; Treasury returns are obtained from CRSP; agency bond returns are calculated based on ICE BofA agency index; corporate bond returns are from Barclays; currency data are downloaded from Datastream; commodity returns are constructed from the GSCI index; REITs returns are obtained from CRSP Ziman REITs indexes; and international stock returns are from MSCI indices in Datastream. Data for different asset classes have different starting dates. The longest data go back to 1963 for stocks and Treasuries. Corporate bond data start from 1973, REITs data start from 1980, and currency data start from 1983. Commodity future returns start from different dates: 1970 for livestock and agriculture, 1973 for precious metal, 1977 for industrial metal, and 1983 for energy. International stock return data start from 1969.

The summary statistics of the average portfolios in each asset class and the cross section of test portfolios are shown in the first two columns of Table 2 and 3. Notably, assets in different asset classes are highly dispersed in average excess returns. For example, the 5 stock portfolios and 3 international stock portfolios have an average excess return of around 7 percent, and the 4 corporate bond portfolio excess returns are 3 percent on average. Treasury excess returns are smaller from 1 to 3 percent. Currency excess returns are dispersed, from -1.81 percent for the lowest interest rate portfolio to 5.56 percent for the highest interest rate portfolio, and the dollar carry portfolio has an average return of 5.34 percent. Commodity futures' excess returns are dispersed as well, from about zero for agriculture and above 7 percent for energy. The equity and hybrid REITs have excess returns even higher than stocks and the mortgage REIT's average excess return is about 5 percent. Stocks, commodity futures and REITs returns are the most volatile, while Treasury returns are the least volatile.

2.1.3 Inflation Shocks

To study the inflation risk, we extract the unexpected component in headline, core, food, and energy inflation from the following VAR system.

$$Y_t = c + AY_{t-1} + \varepsilon_t, \tag{1}$$

where Y_t includes the vector of headline, core, food, and energy inflation, plus the risk-free rate, price-dividend ratio of the aggregate stock market portfolio, and the output gap. The first four elements of ε_t are extracted as the innovations to the four inflation variables in the vector of Y_t. Figure 1 plots the time-series of innovations to the four inflation variables. The estimates of VAR coefficients are reported in Appendix C.1.

The variables that we include in the VAR system are similar to the New Keynesian VAR (see, e.g., Bekaert, Cho, and Moreno (2010)), augmented with the price-dividend ratio. Different from a New Keynesian VAR that attempts to identify the monetary policy shock, we focus on the inflation innovation without attributing the innovation to structural shocks such as TFP, markup, aggregate demand, or other macroeconomic and policy shocks. Therefore, we do not take a stand on the cause of inflation. In this way, we study the overall risk of the unexpected inflation.

The headline inflation shock combines the variation of the three elements. The large spikes in headline inflation shocks are generally driven either by energy or food inflation. For the episodes of 1970s and 1980s, the core inflation is volatile. Before mid 1980s, core inflation tracks the headline inflation closely. After mid 1980s, core inflation is much less volatile than the headline. Energy inflation is a magnitude more volatile than other inflation, especially after the late 1990s. The literature has mostly focused on decreasing inflation volatility (Baele, Bekaert, Cho, Inghelbrecht, and Moreno, 2015), while we find that the trend of inflation volatility is different for core and energy goods. The core inflation volatility is reduced as the monetary policy mainly targets the core inflation, while the energy inflation becomes even more volatile.

We conduct the analysis using other measures of inflation expectations and find robust results. The Survey of Professional Forecasters (SPF) is widely used as a measure of expected inflation (Ang, Bekaert, and Wei, 2007). In our study, we are interested in the expected inflation for core and energy goods separately. The core inflation in SPF only starts from 2007, leaving us a short sample for study, and energy inflation is not included in SPF. Despite the data limitation, we investigate the survey expectation as a robustness check. Since energy inflation is largely unpredictable, we use the expectation of the headline CPI as a proxy for both expected headline inflation and expected core inflation. Besides SPF, we also use expected inflation from the survey of consumers from University of Michigan. The results are robust to different measures of expected inflation and are reported in Appendix C.2.

2.2 Inflation Hedging: Core and Energy

To examine the inflation hedging proprieties, we specify the regression as follows:

$$r_{i,t}^{e} = \alpha_i + \beta_{\pi}^{i} \varepsilon_{\pi,t} + u_{i,t}, \tag{2}$$

where $r_{i,t}^{e}$ is the realized nominal return of asset i in excess of the nominal risk-free rate. β_{π}^{i} represents how much asset i's excess return changes with the shocks to inflation and its components. The shocks $\varepsilon_{\pi.t}$ are extracted from the VAR of equation (1). On the left-hand side, the risk-free rate already reflects changes in inflation expectations, but the realized inflation surprises are not included in the risk-free rate. Therefore, a perfect inflation hedging asset should one-to-one move with the inflation surprise, i.e., $\beta_\pi^i = 1$. If β_π^i is positive but less than 1, the asset is an imperfect inflation hedge.

2.2.1 The Average Portfolios

We start our analysis of inflation risk exposures across asset classes with the 8 average portfolios. Panel A of Table 2 displays the results with respect to the headline inflation shock. The loadings of the U.S. stock market and the world stock market return index on the headline inflation are negative but insignificant. Treasuries, agency bonds, and corporate bonds all have significantly negative headline inflation betas. Currencies and commodity futures hedge against headline inflation with positive betas. The coefficient for the currency portfolio is close to 1, which suggests that the foreign currency is a perfect hedge. The commodity future return moves much more than the headline inflation with a coefficient of 8.59. REITs' headline beta is close to 0 and statistically insignificant. The results in Panel A, to some extent, confirm the conventional wisdom that currencies and commodity futures are inflation-hedging assets.

However, a different picture emerges when we examine core and energy inflation separately. In Panel B of Table 2, we report results with core and energy inflation shocks separately. The U.S. and the world stock market returns load negatively on core and positively on energy inflation, both statistically significant. This result sheds new light on the ambiguous inflation-hedging property of stocks in the literature. The ambiguity in the sign of the stock and REITs' inflation betas is due to the mixture of core and energy inflation. The negative core beta and the positive energy beta add up to an insignificant loading on the headline inflation. Treasuries, agency bonds, and corporate bonds have negative betas with both core and energy inflation. Currencies and commodity futures' hedging properties against headline inflation mainly come from the energy component, while their core betas are negative and insignificant.

The sharp contrast between inflation hedging properties displayed in the two panels in Table 2 shows the importance of decomposing the headline inflation into core and energy components. The average U.S. and international stock, currency, commodity future, and REITs have core and energy betas with opposite signs. The conventional wisdom that stocks, currencies, commodity futures, and real estate are real assets is incomplete: they only hedge against energy inflation. A long position in none of these 8 asset classes can hedge against the core inflation.

2.2.2 The Full Cross Section

We next run regression (2) for the full set of 38 test portfolios and present the results in Table 3. The results are similar with Table 2. Within U.S. stocks, the five industry portfolios have heterogeneous exposures and are negatively exposed to headline inflation except for manufacturing, but only three betas are significant. On the contrary, stocks' core betas are all unambiguously negative and statistically significant, while their energy betas are positive and mixed in statistical significance. The three regional international stocks have similar exposures to inflation and similar average returns with U.S. stocks. All Treasury and agency bond portfolios have negative exposures to headline, core, and energy inflation. Corporate bonds load negatively on core inflation and their energy inflation betas are mixed in sign and mostly insignificant. One possible reason is that the default risk is eased in good economic conditions when the energy price is high.

The dollar carry portfolio has a much more negative core beta than the average currency portfolio. This observation implies that the currency exposures to core inflation depend on the level of interest rate. Carry trade portfolios mostly load negatively on core inflation and positively on energy inflation. High-interest-rate currencies load more negatively on core inflation and more positively on energy inflation - the latter is consistent with high-interest rate currencies being "commodity" currencies (Ready, Roussanov, and Ward, 2017b). Indeed, for commodity futures, energy naturally has a large exposure to energy inflation, and so do other commodity portfolios. But commodity futures do not hedge against the core inflation, with an exception of agriculture whose core beta is positive but statistically insignificant. The three REITs portfolios all have highly significantly negative core betas. The magnitude of these core betas are comparable to those of stocks. REITs are positively exposed to energy inflation, though only the energy beta of the equity REITs is statistically significant.

The findings confirm the conclusion we draw with the 8 average portfolios: exposures to core and energy inflation are fundamentally distinct, especially for the conventional "real" assets.

2.3 Getting to the Core: The Inflation Risk Premium

In the previous section, we show that different asset classes have different exposures to core and energy inflation shocks. In this section, we further explore the cost of hedging against inflation, or the price of these inflation risks.

2.3.1 Inflation Risks Across Asset Classes

Our analysis is based on a factor model of average returns,

$$E(r_{i,t}) = \beta_i' \lambda,$$

where λ is the vector of prices of risks. The first set of risk factors includes the headline inflation only, and the second set of risk factors includes core and energy inflation. We run a Fama-MacBeth cross-sectional regression of average returns onto asset betas to estimate the price of risks and report the results in Table 4. The price of headline risk is statistically insignificant. With core and energy inflation as separate risk factors, core inflation carries a negative price of risk -1.03 that is significant at 99% confidence interval, while the price of energy inflation risk is positive but insignificant. Assets with higher average returns load more negatively on and are hurt more by core inflation. The cross-sectional fit is superb with an R^2 of 0.98. With 38 test portfolios, we utilize more variations in both average returns and asset betas and find a similar price of core inflation risk -1.07 and an even larger t-statistic. The two sets of test portfolios lead to similar estimates of the price of core inflation risk. In Appendix C.3, we report robust results using the GMM method.

The price of risk estimates uncover the second source of difference between core and energy inflation, the cost of exposure. Investors require a compensation of 1.07% of excess return per annum if an asset increases one unit of negative exposure to core inflation. Notably, nearly all assets in our universe have negative betas and are compensated for these exposures. Compensation for energy inflation exposure is the opposite in sign and statistically insignificant. From a hedging perspective, hedging against core inflation is costly, while hedging against energy inflation is free or even rewarded, though indistinguishable from zero.

To visualize the result, Figure 2 plots the average excess returns of the 8 average portfolios (the upper panel) and 38 portfolios (the lower panel) against their model predicted expected excess returns using headline inflation as the only risk factor. The model has a very poor fit for both sets of portfolios. Though the average excess returns for different asset classes vary substantially, the model predicted returns center around zero.

In Figure 3, we plot the average excess returns against the model predicted expected excess returns using both core and energy inflation as risk factors. The cross-sectional fit improves substantially. For 8 average portfolios, the average realized returns and model implied returns line up perfectly. Even with 38 portfolios, these returns line up nicely with an R^2 of about 0.8. The cross-sectional R^2 is higher than asset pricing tests with typical macroeconomic factors, which is a consequence of the multi-asset-class nature of the test. The large R^2 mostly reflects the fit for portfolios across asset classes. The magnitude of average returns within one asset class is similar, and so are the betas.

The sharp contrast highlights the importance of our decomposition in understanding the average returns both within and across asset classes. Negative exposure to core inflation is rewarded with extra return, on average. To see it more clearly, in Figure 4 and 5, we plot the cross-sectional relation between average excess returns and headline, core, and energy inflation betas for the 38 portfolios. Headline betas do not explain average return differences at all: stocks, bonds, and REITs have similar betas but their average returns differ. The core inflation betas line up well negatively with expected excess returns. The average stock and REITs portfolios have the largest negative core betas and the highest returns. Treasuries, agency bonds, and corporate bonds have sizable negative betas and modest returns. Currencies have dispersed core betas which line up with their average returns. Commodity futures have small core exposures and their returns are relatively low. In comparison, the relation between expected excess returns and energy inflation betas is quite noisy as well.

This figure shows the importance of using test portfolios from multiple asset classes. While core and energy betas within the same asset class differ, betas across asset classes are more dispersed and in line with their average excess returns. Dispersed betas improve the power of the statistical test.

2.3.2 Inflation Risks Within Each Asset Class

In the analysis above, we include portfolios from 8 asset classes with roughly equal numbers of portfolios from each asset class, so that none of the asset classes dominates in the price of risk estimates. The limitation is that the number of portfolios are small when we examine inflation risks within each asset class. Therefore, we expand the test portfolios in each asset class. The expanded test portfolios include 35 stock portfolios, 19 Treasury portfolios, 6 agency bond portfolios, 8 corporate bond portfolios, 17 currency portfolios, 8 commodity future portfolios, 11 REITs portfolios, and 7 international stock portfolios. The 35 stock portfolios include 17 industry portfolios and 18 double-sorted portfolios on size and book-to-market, investment, and profitability. The 19 Treasury portfolios include 7 fixed-term portfolios and 12 maturity-sorted portfolios (<6M, 6-12M, 12-18M, 18-24M, 24-30M, 30-36M, 36-42M, 42-48M, 48-54M, 54-60M, 60-120M, >120M). The 6 agency bond portfolios are sorted on maturity: 1-3, 3-5, 5-7, 7-10, 10-15, and >15 years. The 8 corporate bond portfolios are double sorted on credit rating (Aaa-Aa and A-Bbb) and maturity (1-3, 3-5, 5-10, >15 years). The currency portfolios include the 7 portfolios used in the previous analysis plus 4 value-sorted portfolios (Asness, Moskowitz, and Pedersen, 2013; Menkhoff, Sarno, Schmeling, and Schrimpf, 2017), and 6 dollar beta sorted portfolios (Verdelhan, 2018). For commodity futures, we additionally examine the three main components of the precious metal: gold, platinum, and silver. We include 8 additional REITs portfolios: unclassified, diversified, health care, industrial/office, lodging/resorts, residential, retail, and self-storage. For international stocks, besides the three regional portfolios, we sort MSCI country indices into four portfolios according to the import ratio, defined as the sum of complex good imported and basic goods exported divided by total manufacturing production, following Ready, Roussanov, and Ward (2017b).

With the expanded set of test portfolios, we examine the price of inflation risk in each asset class.[2] Table 5 reports the estimates based on portfolios from each asset class, the 8 average portfolios and the full cross section of 38 test portfolios. Strikingly, using test portfolios from different asset classes, we obtain a largely consistent estimate of the price of core inflation risk around -1. In Figure 3, we do see that assets in different asset classes have largely similar slopes between expected excess returns and core inflation betas. Therefore, the core inflation risk is priced consistently both within and across asset classes.

A common concern in testing asset pricing models with macroeconomic factors is the issue of weak factors or weak identification. When betas of test portfolios are similar, we may obtain spuriously large price of risks and cross-sectional R^2. In Panel B of Table 5, we report the p-value of the weak identification test proposed by Kleibergen and Zhan (2020). The p-value indicates the probability of the presence of weak identification and spurious factors. When we include assets from

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[2] For commodity futures, since previous metal consists of gold, platinum, and silver, we only include the precious metal in the estimation.

different asset classes using 8 average portfolios or 38 portfolios, our results can safely pass the weak identification test. Furthermore, the results show the importance of using portfolios across asset classes for robust estimates from an econometric perspective. While the presence of weak factors is soundly rejected in multi-asset-class tests, there are some concerns of weak identification within some of the individual asset classes, especially for commodities, REITs, and international stocks. The lack of identifying power using portfolios within a particular asset class can be shown in Figure 3, in which the betas of portfolios from the same asset class cluster together.

2.3.3 Inflation Factor Mimicking Portfolios

Both core and energy inflation are macroeconomic factors that are not directly traded. It is therefore worthwhile to examine the factor mimicking portfolio returns that represent a in the return space. A factor mimicking portfolio is a linear combination of available asset returns, subject to having the same covariance with the macroeconomic factors as the test assets. Factor mimicking portfolios contain the same pricing information as the macroeconomic factors. We construct the factor mimicking portfolios using the Fama-MacBeth approach.

To construct the factor mimicking portfolios, in every quarter, we regress asset returns on inflation factors to obtain their betas. Factor mimicking portfolio weights are constructed as $(\beta'\beta)^{-1}\beta'$. As a result, a specific factor's mimicking portfolio has unity exposure to the corresponding factor and are orthogonal to other factors.

Panels A through C in Table 6 report the mean, t-statistics, and Sharpe ratios of the factor mimicking portfolios. Columns 1-8 use the expanded portfolios in each specific asset class. Column 9 uses the 8 average portfolios and column 10 uses the 38 portfolios. The average returns of core inflation mimicking portfolios are negative and statistically significant, with the magnitude around -1 percent across all asset classes. The average return of headline inflation mimicking portfolios have different signs for different asset classes and the average return of energy inflation mimicking portfolios are mostly indistinguishable from 0. The properties of factor mimicking portfolios show that the financial market prices core inflation consistently in a stable way within and across asset classes.

2.3.4 The Inflation Hedging Properties of " Real Assets"

Currencies, commodity futures, and real estates are conventionally viewed as inflation-hedging assets, but our previous analysis suggests that they only hedge against energy inflation. It is worth further studying the expanded set of currencies and commodity futures in more detail. In this section, we show that this conclusion applies to all the popular investment portfolios that we consider in the expanded set.

Currencies Currencies are considered to hedge inflation risk because according to the purchasing power parity (PPP), when the U.S. experiences a higher inflation, the purchasing power of dollar declines and the foreign currency appreciates. The literature has established that PPP holds well in the long run (Asness, Moskowitz, and Pedersen, 2013; Menkhoff, Sarno, Schmeling, and Schrimpf, 2017).

Table 7 reports the inflation betas for additional currency portfolios. The value portfolios are sorted on the deviation from PPP (Menkhoff, Sarno, Schmeling, and Schrimpf, 2017). Portfolio 1 contains currencies that are most undervalued relative to their real exchange rates 5 years before. Undervalued currencies will revert back to the fundamental values with expected appreciations. The four value portfolios have positive headline inflation betas but these positive betas mostly come from energy inflation. All four portfolios have negative core betas, though statistically insignificant.

The six dollar-beta-sorted portfolios are constructed following Verdelhan (2018), which are interacted with the sign of the average forward discount. From Portfolio 1 to Portfolio 6, loadings on the dollar exchange rate increase. Currencies that have higher dollar betas have more negative core inflation betas.

Commodities Commodity futures are also conventional inflation-hedging assets. As is shown in Table 3, they only hedge against energy inflation. Kogan and Konstantinovsky (2008) also find that commodities do not hedge core inflation. Among commodities, the precious metal, especially gold, is the most well-accepted assets to preserve value and often thought to hedge against inflation. However, this is not true with the core inflation. Table 7 shows that gold and platinum have positive core inflation betas that are indistinguishable from zero and they only strongly hedge against energy inflation. These precious metal futures have relatively low returns and high volatility. Our results share the same spirit with the finding by Barro and Misra (2016) that gold has a negligible covariance with real macroeconomic risk factors.

REITs Housing is commonly viewed as a real asset class. Leombroni, Piazzesi, Schneider, and Rogers (2020) show that inflation makes housing more attractive than equity. Moreover, a large portion of core inflation is shelter including owners' equivalent rent of residences and rent of primary residence. As of December 2019, the weights of core and shelter in CPI are 79.2% and 33.5%. Therefore, it is natural for investors to consider real estate investment as a good way to hedge against core inflation. Because of the illiquidity of real estate, we focus on REITs.

As shown in Table 2 and 3, REITs behave similarly with stocks: they are strongly negatively exposed to core inflation and positively exposed to energy inflation. The two exposures largely offset with each other so they have insignificant headline inflation betas. Table 7 expands the test assets to REITs in different sectors. The sector portfolios behave quite consistently to the average REITs portfolio in term of headline, core, and energy betas.

Why cannot REITs hedge against core inflation? The specific component of shelter inflation and the average REITs return have a very low correlation of -0.07. Unlike the shelter inflation that reflects the change in single-period rents, REITs returns depend on the future rental income and the discount rate. Because of this difference, REITs behavior is closer to stocks than to that of (shelter) inflation.

TIPS To conclude this section, we consider the asset class that is designed for inflation hedging, the Treasury Inflation Protected Securities (TIPS). TIPS are real assets as their nominal payoffs increase with realized inflation. We use the most recent data available from 2001 and confirm this argument, with results shown in Table 7. TIPS index's headline inflation beta is 0.64, implying that it imperfectly hedges against the headline inflation. However, its exposure to core inflation is 4.54, which is much larger than 1. TIPS return increases by 4.54 percent in response to 1 percent increase in core inflation rate. Surprisingly, the TIPS index does not hedge against energy inflation.

2.4 Time-varying Exposures and Prices of Risk

2.4.1 Time-varying Exposures

The previous analysis explores the unconditional risk exposures and price of risks. In this section, we consider how risk exposures and prices of risk vary over time.

Our analysis of time-varying exposures and risk prices is motivated by the literature that stock-bond correlation changed sign at the turn of the century (Campbell, Sunderam, and Viceira, 2009; Campbell, Pflueger, and Viceira, 2020; Song, 2017). These studies find a structural break in the dynamic behavior of economic fundamentals and/or changes in the monetary policy regime. Following these studies, we split the sample into two subsamples at the structural break: the first from 1963 to 1999 and the second from 2000 to 2019.

Table 8 reports the inflation exposures in the two sub-samples. The stock's core beta is stable over time, but its energy beta increases from zero before the 2000s to being significantly positive after 2000s. Overall, the stock's headline beta switches from negative to positive. The bond exposures to core are more negative before the 2000s, and their exposures to energy are more negative after the 2000s. Currencies and commodity futures hedge against energy inflation more strongly in the second subsample. REITs and international stocks have a pattern similar to U.S. stocks. Panel C provides the p-value of statistical tests on whether the betas change across the two subsamples. Core betas are stable across asset classes, while headline and energy inflation betas show significant structural changes.

To fully explore the time variation, we use a local least square estimator following Adrian, Crump, and Moench (2015). At any time t, the beta estimate follows

$$[\hat{\alpha}(t), \hat{\beta}(t)']' = arg \min_{(\alpha,\beta)} \sum_{i=1}^{n} \frac{K((t_i - t)/h_k T)}{h_k T} (r^e_{i,t_i} - \alpha - \beta' \varepsilon_{\pi,t_i})^2$$

where $K(z) = 1/\sqrt{2\pi} \exp(-z^2/2)$ is a Gaussian density kernel and h_k is a bandwidth. We choose the bandwidth to be 0.05. In Figure 6, the betas slowly evolve over time. Compare our subsample analysis with the kernel estimates, although our simple two-regime approach does not capture all variations, it largely captures the key structural break.

2.4.2 Time-varying Stock-Bond Correlation

The time-variation of stock and bond exposures to core and energy inflation, together with the different volatility patterns can provide some new insights on explaining the changing stock-bond correlation. In our sample, the correlation between the stock market return and the average Treasury return is 0.32 in the first subsample and -0.55 afterward. As we show in Figure 1, core inflation is much more volatile in the early sample and becomes smooth after the mid 1980s. After the late 1990s, energy inflation becomes volatile. Therefore, core inflation's contribution to the overall inflation is lower in the second subsample, and energy inflation's contribution increases.[3] Because of the change of relative contribution and the increase of the energy inflation betas, stocks' exposures to headline inflation switch signs from negative to positive. Since bonds still have negative inflation exposure, the stock-bond correlation turns from positive to negative.

We can attribute the covariance of the stock and bond returns to the covariance driven by inflation shocks and that by the residuals of returns after projecting these returns onto inflation shocks. Before 2000, 35% of the positive covariance is driven by inflation shocks. After 2000, the inflation contribution is 23%. While inflation can explain a sizable amount of the switching signs, the residuals are still correlated potentially due to other previously documented mechanisms, for example, other reasons of real-nominal covariance change (Campbell, Pflueger, and Viceira, 2020) and changing monetary policy forces (Song, 2017).

In sum, core exposure and the price of risk are stable over time. There is an interesting time variation in the relative importance of energy in headline inflation and different assets' energy inflation betas, both of which contribute to changes in their headline inflation exposures. These shifting exposures to inflation risk provide a new economic interpretation of the changing sign of stock-bond return correlation.

2.4.3 Time-varying Prices of Risk

In previous subsections, we find that asset exposures to core inflation do not change significantly in the two subsamples. Moreover, we find the price of core inflation risks is also stable with results

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[3] The attribution of time-varying stock-bond correlation to changing volatility of fundamental economic shocks are similar in spirit to Baele, Bekaert, and Inghelbrecht (2010), who study the switch of regimes in the volatility of different economic shocks.

shown in Panel D of Table 8. The prices of headline risks are insignificant in both samples and energy inflation has a positive price of risk before the 2000s.

Even though the price of core risk does not show a structural change, it might show medium frequency variation with economic conditions. We specify the stochastic discount factor M_{t+1} and the price of risk λ_t as:

$$\frac{M_{t+1} - E_t M_{t+1}}{E_t M_{t+1}} = -\lambda_t \varepsilon_{\pi,t+1},$$

where $\lambda_t = \Sigma_u^{-\frac{1}{2}}(\lambda_0 + \lambda_1 F_t)$. Under this specification, the expected excess return of asset i can be expressed as $E_t R_{t+1}^{i,e} = \beta_i^{'}(\lambda_0 + \lambda_1 F_t)$.

The prices of core and energy inflation risks are both specified as a linear function of economic variable F_t. We choose the 10-year minus 3-month term spread as the conditioning variable and estimate the conditional asset pricing models following the three-step procedure proposed by Adrian, Crump, and Moench (2015). First, we extract the unexpected components of the inflation risk factors. Second, we regress asset returns onto the risk factors as well as lagged values of F_t. Third, λ_0 and λ_1 are constructed using the regression coefficients in the second step. Indeed, the price of core inflation risk decreases with the term spread. The intercept λ_0 is estimated to be -0.94 with a *t*-statistic of -1.70, and the slope λ_1 is estimated to be -0.52 with a *t*-statistic of -1.85. The price of energy inflation risk does not display time variation with the term spread.

2.5 Other Macroeconomic Risk Factors

Previous analysis shows that core inflation risks are priced both within and across asset classes. Does the core inflation risk simply reflect information in other known macroeconomic risks? This idea goes back to Fama (1981) who argues that stocks are negatively exposed to inflation because inflation is countercyclical and stock prices are procyclical.

We consider a set of macroeconomic factors suggested in the asset pricing literature to examine whether the price of core inflation risk is spanned by these macroeconomic factors. These factors include the consumption growth rate, durable consumption growth rate (Yogo, 2006), industrial production growth rate (Chen, Roll, and Ross, 1986), payroll growth rate, unemployment growth rate, and the long run and short-run consumption growth news constructed by Hansen, Heaton, and Li (2008), unfiltered consumption growth (Kroencke, 2017), and capital share growth (Lettau, Ludvigson, and Ma, 2019), intermediary capital ratio (He, Kelly, and Manela, 2017), and the aggregate stock market return (CAPM).

We re-estimate the first-step regression including the macroeconomic factors. The inclusion of macroeconomic factors control for cyclicality and may change the inflation betas and the price of risk estimates. Estimates of the price of risks are reported in Table 9. In untabulated results, we find that the first-stage estimates of inflation betas are robust to the macro factor controls. None of these macroeconomic factors can drive out the negative risk premium of core inflation. The price of core inflation risk estimates remain similar both in magnitude and statistical significance across all specifications, and none of the macroeconomic factors is significantly priced in our portfolios across asset classes. Therefore, the results suggest that core inflation is a good proxy for the stochastic discount factor and provides incremental information beyond existing macroeconomic factors.

2.6 Cash Flow News and Discount Rate News

In this section, we take a closer look at the inflation-hedging properties of stocks. Our previous results indicate that a higher core inflation is negative news for stocks. Is it due to lower cash flow or higher discount rates?

First, we directly study the relation between inflation and future real growth. The literature has documented that inflation has negative impacts on future real growth (Piazzesi and Schneider, 2006; Bansal and Shaliastovich, 2012). Table 10 confirms the inflation nonneutrality that higher headline inflation is associated with lower real growth of GDP, consumption, and dividend. The effects on dividend are the strongest, while those on GDP and consumption are weaker. When we separate core and energy inflation, they have distinct effects. The coefficients are negative and significant for core inflation but are close to zero and insignificant for energy inflation. The core coefficients on dividends are more than twice as large as the headline coefficients. The negative effect of inflation on real growth is concentrated in core inflation.

Next, we study the cash flow (CF) and discount rate (DR) news components of stock returns. We perform standard return decomposition by estimating a VAR(1) system and extracting CF and DR news (Campbell, 1991). The VAR includes the real asset return, the price-dividend ratio, the real risk-free rate, and the headline inflation. We apply the decomposition to estimate the CF and DR news of the aggregate stock market returns, industry portfolio returns, value-sorted porfolio returns, international stock returns, and REITs returns. Then, we estimate the risk exposures of CF and DR news to core and energy inflation in bivariate regressions. Table 11 presents the results. The core exposure of CF news is significantly negative, consistent with the direct evidence on growth measures presented above. The core exposure of DR news is significantly positive. Both news contributes to the negative core exposure of stock returns. Motivated by this fact, the model that we propose in what follows features both CF and DR channels. In contrast, the energy betas of stocks are mainly driven by the DR news. We obtain similar patterns in an analysis of the cross-section of stock portfolios. Interestingly, the cash flow news of growth stocks loads more negatively on core inflation than those of value stocks, while the discount rate news of value stocks loads more on core inflation. The international stocks and REITs share similar patterns to U.S. stocks, except that the energy loadings of REITs mainly come from cash flow news.

One possible mechanism behind the DR channel is monetary policy (Pflueger and Rinaldi, 2020; Cieslak and Pang, 2021; Bianchi, Lettau, and Ludvigson, 2022). When core inflation is high, the Fed raises the policy rate and thus increases the discount rates for all long-duration assets, including stocks. Consistent with this argument, as we show in Appendix C.5, the discount rate exposure of stocks to core inflation is slightly larger in the post-83 period, when monetary policy appears to be more responsive to inflation than in the earlier part of the sample (Song, 2017).

We further explore this monetary policy mechanism by exploiting inflation announcements. Around the short announcement window, it is reasonable to assume that the only market news is the inflation surprise. The inflation surprises are measured as the difference between the median forecast from Bloomberg and the actual announcement. The inflation announcement is scheduled at 8:30 ET before the stock market opening. We examine the high-frequency price reaction in the futures market from 2 minutes before the announcement to 20 minutes after it. Table 12 Panel A shows that fund funds futures increase upon positive inflation surprises. While the policy rate does not change over the window, the market expects a positive policy response in the future FOMC meetings. Comparing the response to core and headline inflation, the response is mostly for the core and insignificant for headline inflation. Panel B reports the stock market reactions. First, futures prices fall with inflation surprises, consistent with the negative core beta documented in Section 2.2. The reaction to core surprises is three times larger than the headline surprises, again highlighting the importance of looking at core. Second, consistent with the DR channel, increases in the Fed Funds Futures are associated with declines in stock returns. This evidence confirms that the stock market response to inflation news is partially through the monetary policy response. Third, controlling for the Fed Funds Futures change, the effect of core inflation surprises are still significant with a slightly smaller magnitude. Therefore, stocks react to inflation above and beyond its role in driving the immediate monetary policy response that is anticipated by the stock market.

2.7 Expected and Unexpected Inflation

In the previous analysis, we examine what assets hedge against unexpected inflation shocks. When investors consider inflation hedging, they may not necessarily distinguish the expected and unexpected components of inflation and simply care about the comovement between asset returns and inflation levels (Bekaert and Wang, 2010). In this section, we consider this setting and regress the realized excess return of asset i onto the realized level of inflation (headline, core, and energy) as follows:

$$r_{i,t}^{e} = \alpha_i + \beta_{\text{level}}^{i} \pi_t + u_{i,t}, \tag{3}$$

where β_{level}^{i} represents the comovement between excess return and inflation level.

Further, we decompose the level of inflation into the expected and unexpected components, with the expected inflation defined as $E_{t-1}\pi_t = \pi_t - \varepsilon_{\pi,t}$. We run the following regression:

$$r_{i,t}^{e} = \alpha_i + \beta_{e}^{i} \left(E_{t-1}\pi_t\right) + \beta_{u}^{i} \varepsilon_{\pi,t} + u_{i,t}. \tag{4}$$

In this specification, β_e^i represents how much asset i's excess return changes with the expected inflation. Since the risk-free rate in principle incorporates the change in expected inflation, a zero β_e^i indicates that asset i has the same hedging property against expected inflation as the risk-free rate.

Table 13 Panel A reports estimates of equation (3) and (4) for the headline inflation. β_{level} are qualitative similar to β_u, but the magnitude of β_{level}'s for the fixed-income securities and commodity futures are smaller. The overall β_{level} includes both expected and unexpected inflation hedging. The expected inflation exposure β_e are not significantly different from zero except for corporate bonds, which implies that most asset classes can hedge expected inflation as well as the risk-free rate. These decomposition results illustrate that hedging expected and unexpected inflation are distinct phenomena.

Panel B reports the estimates for core and energy inflation. The results of core inflation share the same message as the headline inflation that β_{level} mixes the expected and unexpected inflation. All the assets we consider hedge against the expected core inflation, while it is very difficult to hedge against unexpected core inflation. We do not include the energy inflation expectation as energy inflation is fairly transitory.

In Appendix C.6, we report the asset pricing test results for the 8 average portfolios with respect to shocks to expected core inflation, which is constructed as $A\varepsilon_{\pi,t}$ and it has a high correlation of 0.90 with the core inflation shock. All results are very close to those with core inflation shocks.

Inflation hedging can be of importance for long-term investors over longer horizons. For example, a long-term investor may not care about hedging against quarter-over-quarter inflation. Instead, they want to hedge against inflation over a few years. In Table 14, we repeat our analysis of inflation hedging over a longer horizon of 8 quarters. The longer-horizon expected inflation is extracted from the VAR as detailed in Appendix C.7.

Our longer-horizon analysis leads to a similar conclusion that the excess returns of the 8 asset classes we consider hedge against expected core inflation almost as well as the risk-free rate. The betas with respect to unexpected core inflation are slightly smaller for all assets except for commodities but remain significant . The energy inflation betas change little, since energy inflation is largely unpredictable. This result shows that even at the medium-term horizon of 2 years, our conclusions regarding inflation-hedging properties of different asset classes still apply, at least for the unexpected component of core inflation.

2.8 Price Stickiness

What is the economic mechanism that makes core and energy inflation different? Motivated by their different persistence, we conjecture that price stickiness may be one underlying reason. Core and energy goods and services differ greatly in price stickiness. Energy prices are flexible while core good prices are more sticky. To verify this conjecture, we categorize goods and services into flexible and sticky prices and examine the properties of flexible and sticky inflation. The data are constructed by the Federal Reserve Bank of Atlanta using information on the frequency of price changes of various spending categories. The sticky price inflation is a weighted basket of sticky-price goods. These goods account for 70% of the whole basket and their prices change relatively slowly every 5 to 26 months. The flexible price goods account for 30% and their prices change every 1 to 4 months. The sticky price items are mostly core goods, such as personal care fees, motor vehicle fees, water, sewer, and trash collection services, medical care services, etc. The flexible price items include motor fuel, car and truck rental, fresh fruits and vegetables, etc, which are mostly food and energy goods. The sticky and core inflation share similar properties. Shocks to the core and sticky inflation have a correlation of 0.85, and shocks to the energy and flexible inflation have a correlation of 0.91. The correlation between sticky (flexible) and energy (core) inflation shocks is as low as 0.12 (0.23).

Moreover, we confirm that the asset exposures to sticky and flexible inflation risks and their prices of risk resemble the properties of core and energy inflation. Table 15 reports the results of inflation exposures for 8 average portfolios and the price of risk estimates for both 8 and 38 portfolios. These results support our conjecture that price stickiness may be the economic mechanism that leads to the differences in core and energy inflation. Motivated by this supportive evidence, we develop a two-sector New-Keynesian model, in which the core goods have higher price stickiness than energy goods and core inflation negatively comoves with economic conditions.

3 The Model

In this section, we propose a two-sector New Keynesian model that helps us understand the empirical findings in the previous section. The model is intended to be stylized and includes a minimum set of ingredients to accommodate our empirical findings. For brevity, we only present essential information about the model in the main text. Details are relegated to Appendix A.

3.1 Model Setup

3.1.1 Households

Representative households derive utility from a consumption basket C_t, which consists of the consumption of core goods $C_{c,t}$ and energy goods $C_{e,t}$. The consumption basket is the aggregation of the two goods with constant elasticity of substitution (CES): $C_t = [\alpha_c C_{c,t}^{(\phi-1)/\phi} + (1 - \alpha_c)(\exp(\delta_t) C_{e,t})^{(\phi-1)/\phi}]^{\phi/(\phi-1)}$. α_c is the weight on core consumption. ϕ is the elasticity of substitution between core and energy goods. A higher value of ϕ means that core and energy goods are more substitutable. δ_t is an exogenous shock to the relative demand of energy. Larger δ_t means a relatively higher demand for the energy good.

Households have CRRA utility over the consumption basket and disutility from labor supply. They maximize their lifetime utility subject the budget constraint as

$$\max_{C_{c,t},C_{e,t},\eta_t,N_t} E\sum_{t=0}^{\infty}\beta^t\left[\frac{C_t^{1-\gamma}-1}{1-\gamma}-\frac{N_t^{1+\varphi}}{1+\varphi}\right],$$

$$s.t.: P_{h,t}P_tC_t+\eta_t=W_tN_t+P_{e,t}P_tC_{e,t}+\eta_{t-1}^{'}R_t.$$

The core good is the numeraire and its real price equals 1. $P_{h,t}$ is the real price of the consumption basket (headline price), and $P_{e,t}$ is the real price of the energy good. P_t is the nominal price of the core good, W_t is the nominal wage. η_t is a vector of the asset holding value, and R_t is a vector of corresponding asset returns. The specification includes a generic set of assets, including the nominal risk-free rate asset whose return is $1+i_{t-1}$ and the return of a claim to all firms in the economy.

3.1.2 Production

The core good consists of a continuum of varieties $C_{c,t}(i)$, which are aggregated through a CES aggregator: $C_{c,t}=(\int_0^1 C_{c,t}(i)^{(\varepsilon_t-1)/\varepsilon_t}di)^{\varepsilon_t/(\varepsilon_t-1)}$. ε_t is the elasticity of substitution across varieties. Each variety $i\in[0,1]$ of core goods is produced by a firm in a monopolistic competitive environment. The desired markup $\exp(\mu_t)\equiv\varepsilon_t/(\varepsilon_t-1)$ fluctuates exogenously. This is a modeling device to capture the variation of inflation that are independent of other real, financial, and policy shocks. In our model, the markup shock is the main driver of core inflation. This specification follows the finding in the New Keynesian DSGE literature (for example, Smets and Wouters (2007)) that markup shocks are the dominant drivers of inflation, while other shocks (e.g. total factor productivity, investment, monetary and fiscal policy, and financial shocks) explain only a minor fraction of inflation. Therefore we abstract away these other shocks in our model.

The production technology of each variety of core good is $C_{c,t}(i)=N_t(i)^{1-\alpha}$. The production technology has decreasing returns to scale. In each period, firms face price rigidity and may adjust their prices with probability $1-\theta$. Firms set optimal price facing the demand schedule implied by monopolistic producers. The quantity of energy good, $C_{e,t}$, is exogenously endowed in each period.

3.1.3 Monetary Policy

The central bank follows the Taylor rule:

$$i_t = \bar{i} + \phi_\pi \pi_t. \tag{5}$$

The nominal interest rate responds to current inflation of the core goods. $\phi_\pi > 1$ implies that when the current inflation is high, the central bank raises the nominal rate more than one-for-one to fight against the inflation, and thus the real interest rate increases. Through the Taylor rule, asset prices will incorporate expectations of future monetary policy responses to inflation. To focus on inflation, we omit the response to output.

3.1.4 Exogenous Processes

There are three exogenous processes: the markup, energy endowment, and relative energy demand. All three exogenous shocks follow first-order autoregressive processes.

$$\mu_t = (1-\rho_\mu)\bar{\mu} + \rho_\mu \mu_{t-1} + \sigma_\mu \varepsilon_{\mu,t}, \tag{6}$$

$$\log C_{e,t} \equiv c_{e,t} = (1-\rho_e)\bar{c}_e + \rho_e c_{e,t-1} + \sigma_e \varepsilon_{e,t}, \tag{7}$$

$$\delta_t = \rho_\delta \delta_{t-1} + \sigma_\delta \varepsilon_{\delta,t}. \tag{8}$$

To sum up, core goods and energy goods are different in four dimensions. First, core goods are produced by a variety of monopolistic competitive firms, while energy goods are endowed and the market is perfectly competitive. Second, the producers of core goods face price rigidity, which is specified later and energy prices are flexible. Third, core and energy inflation are driven by distinct demand and supply factors. Lastly, monetary policy responds to the core inflation, but not the energy inflation.

3.2 Equilibrium Characterization

We approximate the model with log-linearization and solve for the real variables analytically. Lowercase letters refer to the log of each variable. Macroeconomic variables are deviations from the deterministic steady state.

We make the following assumption on the parameters of the model.

Assumption 1 *The elasticity of substitution between core and energy is greater than unity, but less than the intertemporal elasticity of substitution, i.e.* $1 < \phi < \frac{1}{\gamma}$.

Results collected in the lemmas below are useful for understanding the model.

Lemma 1 *(Energy price) The real energy price is proportional to the ratio of core and energy goods, adjusted by the relative demand shock, i.e.:*

$$p_{e,t} = \left(1 - \frac{1}{\phi}\right)\delta_t + \frac{1}{\phi}(c_{c,t} - c_{e,t}). \tag{9}$$

Intuitively, energy demand raises energy price and energy supply lowers energy price.

Lemma 2 *(Consumption basket and headline price) Households' consumption basket is a weighted average of core and energy consumption, adjusted by the energy demand shock. The real headline price is proportional to real energy price after the adjustment of energy demand shock:*[4]

$$c_t = \hat{\alpha}_c c_{c,t} + (1 - \hat{\alpha}_c)(c_{e,t} + \delta_t), \tag{10}$$

$$p_{h,t} = \frac{1 - \hat{\alpha}_c}{\phi}(c_{c,t} - c_{e,t} - \delta_t). \tag{11}$$

Lemma 2 states that core and energy consumption and the energy demand shock can effectively raise the consumption basket. Given the consumption of core and energy goods, the energy demand shock lowers the headline price because less energy is needed to achieve the same level of the consumption basket.

Lemma 3 *(Marginal utility of core goods) Under Assumption 1, the marginal utility of core goods,* $MU_{cc,t} = \alpha_c C_t^{\frac{1}{\phi}-\gamma} C_{c,t}^{-\frac{1}{\phi}}$*,increases with energy supply and demand* $c_{e,t}$ *and* δ_t.

Assumption 1 states that core and energy goods are complementary, so that the marginal utility of core goods increases with the energy good consumption. $\phi < 1/\gamma$ implies that the core goods' marginal utility increase is strong enough to dominate the incentive to postpone consumption into the future. The following proposition summarizes the core output and inflation's loadings on the three state variables.

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[4] $\hat{\alpha}_c = \frac{\alpha_c}{\alpha_c + (1-\alpha_c)\left(\frac{C_e}{C_c}\right)^{\frac{\phi-1}{\phi}}}$ is a constant dependent on steady-state consumption.

Proposition 1 *(Core output and inflation) The core consumption and core inflation can be expressed as linear functions of the three state variables: markup, energy supply, and energy demand.*

$$c_{c,t} = c_{c,\mu}\mu_t + c_{c,e}c_{e,t} + c_{c,\delta}\delta_t, \pi_t = \pi_\mu\mu_t + \pi_e c_{e,t} + \pi_\delta\delta_t.$$

The signs of the loadings are

$$c_{c,\mu} < 0, c_{c,e} > 0, c_{c,\delta} > 0,$$

$$\pi_\mu > 0, \pi_e > 0, \pi_\delta > 0.$$

The formulae for these coefficients are shown in Appendix B.1. The signs of core output and inflation loadings on the markup shock $c_{c,\mu}, \pi_\mu$ are similar with standard New Keynesian models. A positive markup shock increases the price charged by core firms, so the nominal price of the core good increases and core production is reduced due to the downward sloping demand curve.

The loadings of core output and inflation on the energy supply shock reflect the shock's effects on core supply and demand. As in Lemma 3, the increased energy supply increases the marginal utility of core and thus pushes down the wage and increases the aggregate supply of core goods. On the demand side, the increased the marginal utility of the core good raises the core demand. Therefore, energy supply shock is expansionary, i.e., $c_{c,e} > 0$. The loading of core inflation on the energy supply shock depends on the relative importance of demand and supply forces mentioned above. In Appendix B.1, we show that the demand force dominates and core inflation increases with energy supply, i.e., $\pi_e > 0$.

We notice that the energy supply and demand shocks have identical effects on the core output and inflation from Lemma 2. A positive energy demand shock increases core output and inflation. $c_{c,\delta} > 0, \pi_\delta > 0$.

3.3 Energy Inflation and Headline Inflation

The nominal energy inflation is the change of real price of energy plus the inflation of core goods.

$$\pi_{t+1} + \Delta p_{e,t+1} = -p_{e,t} + \left(\pi_\mu + \frac{1}{\phi}c_{c,\mu}\right)\mu_{t+1} + \left(\pi_e + \frac{1}{\phi}(c_{c,e} - 1)\right)c_{e,t+1} + \left(\pi_\delta + \frac{1}{\phi}(c_{c,\delta} + \phi - 1)\right)\delta_{t+1}. \tag{12}$$

Since $\pi_\mu, c_{c,\mu} < 0$, $\pi_\mu + \frac{1}{\phi}c_{c,\mu} < 0$. A positive markup shock lowers energy inflation.

The energy demand and supply shocks affect energy inflation both directly and indirectly. The direct effect is negative for energy supply shock and positive for energy demand shock, holding core output and inflation fixed. The indirect effects work through core output and inflation: both shocks raise core output and inflation, which in turn translates into higher energy inflation. The direct effect dominates the indirect effect when the share of energy good $1-\hat{\alpha}_c$ is relatively small.

The headline inflation is expressed as

$$\pi_{h,t} = \pi_t + \Delta p_{h,t} = \pi_t + \frac{1-\hat{\alpha}_c}{\phi}\Delta(c_{c,t} - c_{e,t} - \delta_t).$$

The exposure of headline inflation to the markup shock is $\pi_\mu + \frac{1-\hat{\alpha}_c}{\phi}c_{c,\mu} < 0$. The sign of headline inflation's loadings on energy supply and demand shocks π_e and π_δ is ambiguous, depending on how the energy supply and demand shocks affect core inflation and consumption.

3.4 Asset Pricing Implications

In this section, we use the Euler equation to price assets including core stocks, nominal bonds, currencies, and commodity futures. As core inflation is mainly driven by the markup shock, we interpret the loadings of SDF and asset returns on the markup shock as the loadings on core inflation. For energy inflation shocks, it is a weighted average of loadings on energy demand and supply shock, which we will show explicitly.

3.4.1 Stochastic Discount Factor (SDF) and the Price of Risk

We consider the SDF in real terms in the unit of consumption basket.

Proposition 2 *(The SDF and the price of risk) The log SDF m_{t+1} is expressed as*

$$m_{t+1} = m_\mu(1-\rho_\mu)\mu_t + m_e(1-\rho_e)c_{e,t} + m_\delta(1-\rho_\delta)\delta_t - \lambda_\mu\sigma_\mu\varepsilon_{\mu,t+1} - \lambda_e\sigma_e\varepsilon_{e,t+1} - \lambda_\delta\sigma_\delta\varepsilon_{\delta,t+1}$$

where the signs of the coefficients are

$$m_\mu < 0, m_e > 0, m_\delta > 0,$$

$$\lambda_\mu < 0, \lambda_e > 0, \lambda_\delta > 0.$$

The proof of this proposition is in Appendix B.2. When there is a positive markup shock, core consumption decreases and the marginal utility of consumption increases. Therefore, core inflation has a negative price of risk.

For either energy demand or supply, a positive shock increases the marginal utility of core consumption and lowers the relative price of the consumption basket. Therefore, both shocks have identical positive prices of risk. Ignoring the indirect effect of energy shocks on the energy good price through core consumption and core inflation, we obtain the following expression for the price of energy inflation risk:

$$\lambda_{energy} \propto -\frac{1}{\phi}\lambda_e\sigma_e^2 + \frac{\phi-1}{\phi}\lambda_\delta\sigma_\delta^2.$$

There are two offsetting forces determining the price of energy inflation risks, one from energy supply $-\frac{1}{\phi}\lambda_e\sigma_e^2$, and the other from energy demand $\frac{\phi-1}{\phi}\lambda_\delta\sigma_\delta^2$. If the energy demand dominates energy supply in driving the energy inflation, the price of energy inflation is positive, which is consistent with our empirical evidence though it is not statistically significant. Otherwise, the price of energy inflation is negative. The offsetting forces helps us understand why it is hard to identify the price of energy inflation risk in the data.

When we discuss asset returns, especially bonds, we may use the nominal SDF. The nominal SDF $m_{t+1}^{\$}$ is expressed as $m_{t+1}^{\$} = m_{t+1} - (\pi_{t+1} + \Delta p_{h,t+1})$, where $\pi_{t+1} + \Delta p_{h,t+1}$ is the headline inflation. If inflation is neutral and does not affect the real SDF, the nominal price of inflation (and all its components) should be 1. Any deviation is caused by the effect of inflation shocks (both core and energy) on the real economic quantities.

3.4.2 Core Stock Returns

A core stock is a claim to the core good producers' dividend, which equals core output net of labor cost. We express the return of the core stock as a log-linear function of the three exogenous state variables.

Proposition 3 *(Core stock return) When the steady state level of the markup* $\bar{\mu}$ *is sufficiently large, i.e.,* $\exp(-\bar{\mu}) < \frac{1}{(1-\alpha)(\gamma+\frac{\varphi+1}{1-\alpha})}$, *the real core stock return* $r_{s,t+1}$ *is expressed as an affine linear function of the three state variables:*

$$r_{s,t+1} = r_{s0} + r_{s,\mu}\mu_{t+1} + r_{s,e}c_{e,t+1} + r_{s,\delta}\delta_{t+1}. \tag{13}$$

We can determine the signs of the coefficients as follows:

$$r_{s,\mu} < 0, r_{s,e} > 0, r_{s,\delta} > 0.$$

Core stock returns decrease with a positive markup shock, a negative energy supply shock and a negative energy demand shock.

The proof is in Appendix B.3. The loadings of core stock returns on the three shocks are intuitive. Core stock returns decline with a positive markup because the markup shock is contractionary. For energy demand and supply shocks, both of them are expansionary and increase core stock returns. Energy demand and supply shocks affect energy inflation in opposite ways. As in the previous section, we can write the loadings of core stock returns on energy inflation as

$$\beta_{s,energy} \propto -\frac{1}{\phi} r_{s,e}\sigma_e^2 + \frac{\phi-1}{\phi} r_{s,\delta}\sigma_\delta^2.$$

The sign of the loadings of core stock return on energy inflation depends on whether supply or demand shock is dominant in determining the energy inflation. Our empirical evidence of positive $\beta_{s,energy}$ suggests that energy demand shocks are the main drivers of energy prices.

3.4.3 Nominal Bond Returns

We next calculate the nominal return to default-free long-term bonds. To calculate the long-term nominal bond returns, we need to use the nominal SDF $M_{t+1}^{\$}$. Here, we consider a two-period bond for an analytical solution as in the following proposition.

Proposition 4 *(Long-term bond return) Denote* $r_{b,t+1}$ *be the* $t+1$*-realized holding-period return of a two-period long-term bond issued at time* t*. It is an affine linear function of the three state variables*

$$r_{b,t+1} = r_{b,0} + r_{b,\mu}\mu_{t+1} + r_{b,e}c_{e,t+1} + r_{b,\delta}\delta_{t+1}. \tag{14}$$

The loadings and their signs are as follows:

$$r_{b,\mu} < 0, r_{b,e} < 0, r_{b,\delta} < 0.$$

The nominal bond return decreases with a higher markup, a higher energy supply, and a higher energy demand.

The proof is in Appendix B.4. The exposure of the long-term bond return to all three shocks are negative. When there is a positive markup shock, consumption goes down and bond price decreases.

Moreover, the contemporaneous and future inflation makes nominal bond less attractive so that the bond price drops.

The positive energy shock (either demand or supply) has two effects on the nominal bond return. First, it raises the consumption of the basket and increases the bond price. Second, it raises the inflation expectations and decreases the bond price. As shown in the proof in Appendix B.4, the second effect dominates, so that nominal bond return decreases with a positive energy supply or demand shock.

As in our analysis for the SDF and core stocks, bond returns' energy inflation beta is a weighted average of their betas on energy supply and demand shocks. When energy demand shocks are dominant, nominal bond returns comove negatively with energy price, which is consistent with the empirical evidence. The bond market evidence again suggests that energy demand is the dominant driver of energy inflation over the entire sample. However, as documented in Section 2.4, over the first part of our sample (pre-1999) the energy beta of stocks is negative, suggesting that the supply shocks were more important during the 20th century subsample, while the demand channel emerged as the driving force in the first decades of the 21st century.

3.4.4 Currency Returns

Next, we consider the foreign currency returns. Assume the financial market is complete, the nominal exchange rate is equal to $\Delta q_{t+1} = m^{*\$}_{t+1} - m^{\$}_{t+1}$. Holding the foreign SDF fixed, foreign exchange rate moves in the opposite direction to $m^{\$}_{t+1}$. Therefore we have the following proposition:

Proposition 5 *(Foreign currency return) Denote $r_{fx,t+1}$ the nominal return to a long position in the foreign currency that is realized at time $t+1$. We can write it as an affine linear function of the three state variables:*

$$r_{fx,t+1} = r_{fx,0} + r_{fx,\mu}\mu_{t+1} + r_{fx,e}c_{e,t+1} + r_{fx,\delta}\delta_{t+1}.$$

where $r_{fx,\mu} < 0$, and the sign of $r_{fx,e}$ and $r_{fx,\delta}$ are ambiguous, depending on the relative strength of three forces: the change of real SDF m, the change of headline relative price p_h, and the inflation rate of core goods π.

Proof is in Appendix B.5. A positive markup shock increases the real SDF and depreciates the foreign currency, while a nominal inflation appreciates the foreign currency. Combine the two effects, the real exchange rate effect dominates the nominal effect so that the foreign currency loads negatively on the markup shock. A positive energy shock has multiple effects on foreign currencies. First, the real SDF decreases ($\lambda_e > 0$) and the real foreign exchange rate appreciates. Second, inflation rate of core goods increases with the energy shock $\pi_e > 0$. Which effect dominates depends on the parameter values.

3.4.5 Commodity Future Returns

Finally, we derive the price of commodity futures and its exposure to the three fundamental shocks (the only good that can be described as a traded commodity - i.e. a non-differentiated good that is traded in a competitive spot market - is the energy good). Denote f_t the log price of commodity future.

Proposition 6 *(Commodity future returns) The price of commodity futures is expressed as* $f_{t+1} = f_0 + f_\mu \mu_{t+1} + f_e c_{e,t+1} + f_\delta \delta_{t+1}$*, where* $f_e < 0, f_\delta > 0$*, and* f_μ *is ambiguous.*

Proposition 6 is very intuitive. Under both energy supply and demand shocks, commodity future returns move in the same direction as the energy inflation so that commodity futures have unambiguously positive loadings on energy inflation shocks. The model also implies that commodity futures' exposures to the markup shock are ambiguous. On one hand, a markup shock makes the core good more scarce and the real energy price lower. On the other hand, the markup shock increases the nominal price level. In the data, the exposure of commodity futures to core inflation is very poorly estimated and it is hard for us to determine the sign of the core inflation exposure, which suggests that the same countervailing forces might be at work empirically.

3.5 Extension: Heterogeneous Agents

One limitation of the stylized model is that consumption-based CAPM holds and the effect of core inflation on asset prices should be spanned by the consumption growth. However, in the empirical section, we find a significant core inflation risk premium even after controlling for various consumption risk measures.

We extend our model by introducing two agents: workers and shareholders. Workers supply labor and only trade goods in the spot market without access to the financial market. Shareholders hold and price financial assets. As a result, inflation affects asset prices through shareholders' consumption but is disconnected from the aggregate consumption. We show in Appendix D that the propositions on stochastic discount factors and asset risk exposures are preserved in the extended model. Besides resolving the consumption measurement issue, the extended model implies a theoretical possibility that inflation has heterogeneous effects on different agents.

4 Conclusion

This paper provides new insights into the nature of inflation risks by decomposing inflation into core and energy components. These components have sharply different asset pricing properties. Conventional inflation "hedges" such as stocks, currencies, commodities, and REITs only hedge energy inflation, but not the core. The core inflation carries a negative risk premium, which is consistently estimated within and across asset classes. A two-sector New Keynesian model is developed to understand these empirical results qualitatively. Our inflation risk decomposition also sheds new light on the changing stock-bond correlation: as energy prices have been both volatile and largely dominated by demand shocks post-2000, stocks have appeared to be positively correlated with inflation, and hence negatively with bond returns. In an environment where core inflation volatility is dominant and energy prices are mostly driven by supply shocks - a regime observed during much of the 20th century - the stock-bond correlation is positive, as both are affected negatively by (core) inflation.

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Table 1: Summary Statistics of Inflation

A. Summary

	Mean	S.D.	Autocorr
Headline	3.76	3.24	0.60
Core	3.75	2.66	0.79
Food	3.75	4.04	0.43
Energy	4.01	19.52	0.04

B. Regression

	β	*s.e.*
Core	0.71	0.01
Food	0.20	0.01
Energy	0.09	0.00

C. Correlation

	Headline	Core	Food	Energy
Headline	1.00			
Core	0.80	1.00		
Food	0.60	0.44	1.00	
Energy	0.69	0.20	0.17	1.00

Notes: This table provides summary statistics of the headline inflation and its three components, core, food, and energy inflation. Data are quarterly from 1963Q3 to 2019Q4. All numbers are annualized. Panel A reports the mean, standard deviation, and autocorrelation of each series. Panel B reports the regression coefficients of headline inflation on core, food, and energy inflation. Panel C reports the correlation matrix.

Table 2: Average Portfolio Exposures to Inflation Risks

			A. Headline		B. Core and Energy			
	Mean	S.D.	headline β	t-stat	core β	t-stat	energy β	t-stat
Stock	6.80	16.79	-1.33	(-1.38)	-5.60	(-3.69)	0.21	(1.81)
Treasury	2.07	6.90	-2.53	(-7.06)	-2.51	(-4.27)	-0.20	(-4.57)
Agency	2.44	5.10	-1.62	(-5.42)	-2.25	(-4.28)	-0.09	(-2.75)
Corporate	3.08	6.39	-1.60	(-4.38)	-2.98	(-4.91)	-0.05	(-1.08)
Currency	1.76	7.05	1.04	(2.02)	-1.04	(-0.65)	0.13	(2.54)
Commodity	4.47	21.90	8.59	(7.53)	-0.07	(-0.04)	1.10	(8.21)
REIT	7.96	17.46	0.31	(0.27)	-6.54	(-3.30)	0.31	(2.48)
Intl Stock	6.09	16.53	-1.20	(-1.23)	-5.78	(-3.74)	0.19	(1.70)

Notes: This table reports the regression results of the specification $r^e_{i,t} = \alpha_i + \beta^i_\pi \varepsilon_{\pi,t} + u_{i,t}$ for 8 average portfolios in each asset class. $r^e_{i,t}$ is the return of asset i in excess of the risk-free rate in the US. $\varepsilon_{\pi,t}$ is the shock to respective inflation extracted from the VAR system. Panel A uses headline inflation shock as the risk factor. Panel B uses core and energy inflation jointly as risk factors. The t-statistics are in the parentheses. The first two columns report the mean and standard deviation of returns in each row. All returns and inflation variables are annualized and span 1963 to 2019 at the quarterly frequency.

Table 3: Asset Return Exposure to Inflation Risks

			A. Headline		B. Core and energy			
	Mean	S.D.	headline β	t-stat	core β	t-stat	energy β	t-stat
Stock								
Consumer	7.83	17.70	-2.62	(-2.61)	-6.34	(-3.97)	0.06	(0.48)
Manufacturing	6.65	15.49	0.32	(0.35)	-4.20	(-3.02)	0.36	(3.39)
High Tech	7.31	20.29	-1.17	(-1.00)	-6.07	(-3.29)	0.26	(1.86)
Health	8.67	17.80	-2.73	(-2.70)	-6.30	(-3.91)	0.04	(0.34)
Others	7.27	20.09	-2.38	(-2.08)	-7.40	(-4.09)	0.17	(1.22)
Treasury								
1-year	0.96	1.87	-0.56	(-5.60)	-0.84	(-5.20)	-0.03	(-2.20)
3-year	1.19	3.16	-0.97	(-5.70)	-1.44	(-5.26)	-0.05	(-2.24)
5-year	1.93	5.86	-1.85	(-5.90)	-2.21	(-4.34)	-0.13	(-3.28)
7-year	2.35	6.97	-2.33	(-6.31)	-2.46	(-4.08)	-0.18	(-3.89)
10-year	2.19	8.29	-2.68	(-6.07)	-3.10	(-4.30)	-0.19	(-3.40)
20-year	2.95	11.32	-4.16	(-7.05)	-3.79	(-3.92)	-0.35	(-4.82)
30-year	2.94	13.29	-5.18	(-7.60)	-3.72	(-3.33)	-0.51	(-6.00)
Agency Bond								
1-5 year	1.83	3.94	-1.17	(-4.99)	-1.90	(-4.66)	-0.05	(-2.03)
5-10 year	3.58	5.20	-1.48	(-3.89)	-0.26	(-0.21)	-0.14	(-3.70)
10-15 year	3.62	8.64	-2.84	(-5.69)	-3.71	(-4.25)	-0.18	(-3.10)
>15 year	4.76	10.38	-3.42	(-5.72)	-3.63	(-3.44)	-0.26	(-3.66)
Corporate Bond								
1-3 year	2.26	3.21	-0.48	(-2.44)	-1.56	(-4.69)	0.02	(0.70)
3-5 year	2.93	4.89	-0.84	(-2.78)	-2.14	(-4.17)	0.00	(0.06)
5-10 year	3.61	6.91	-1.25	(-2.93)	-2.98	(-4.05)	-0.01	(-0.26)
>15 year	4.27	10.13	-2.85	(-4.98)	-4.47	(-4.66)	-0.13	(-1.91)
Currency								
Dollar-carry	5.34	8.82	-0.98	(-1.52)	-4.17	(-2.08)	0.00	(-0.04)
Carry-1	-1.81	7.94	0.33	(0.57)	-0.52	(-0.28)	0.06	(0.95)
Carry-2	-0.25	7.47	1.60	(2.99)	1.72	(1.03)	0.14	(2.55)
Carry-3	1.12	7.27	1.02	(1.92)	-0.04	(-0.02)	0.11	(2.02)
Carry-4	2.53	8.20	0.45	(0.74)	-2.50	(-1.34)	0.10	(1.60)
Carry-5	3.43	8.76	1.44	(2.28)	-1.28	(-0.65)	0.19	(2.94)
Carry-6	5.56	10.10	1.38	(1.87)	-3.62	(-1.60)	0.20	(2.72)
Commodity								
Livestock	2.70	16.99	1.24	(1.24)	-1.09	(-0.66)	0.15	(1.22)
Industrial metal	4.23	25.69	4.73	(2.98)	-1.07	(-0.39)	0.66	(3.66)
Precious metal	3.41	20.96	3.28	(2.65)	-0.22	(-0.11)	0.43	(2.96)
Energy	7.26	36.93	16.51	(7.05)	-0.76	(-0.11)	1.78	(7.54)
Agriculture	0.28	22.24	4.20	(3.28)	2.06	(0.96)	0.26	(1.66)
REIT								
Equity	8.31	17.87	0.72	(0.61)	-6.48	(-3.20)	0.35	(2.77)
Mortgage	4.73	21.15	-2.25	(-1.63)	-8.61	(-3.56)	0.04	(0.25)
Hybrid	8.20	20.31	-1.05	(-0.79)	-6.14	(-2.60)	0.12	(0.79)
International Stock								
NorthAmerica	6.82	16.36	-0.92	(-0.96)	-5.47	(-3.57)	0.23	(2.02)
Europe	6.60	18.63	-0.93	(-0.85)	-6.09	(-3.48)	0.20	(1.56)
FarEast	7.01	22.66	-1.33	(-0.99)	-5.05	(-2.32)	0.15	(0.93)

Notes: This table reports the regression results of the specification $r^e_{i,t} = \alpha_i + \beta^i_\pi \varepsilon_{\pi,t} + u_{i,t}$ for 38 portfolios in each asset class. $r^e_{i,t}$ is the return of asset i in excess of the risk-free rate in the US. $\varepsilon_{\pi,t}$ is the shock to respective inflation extracted from the VAR system. Panel A uses headline inflation shock as the risk factor. Panel B uses core and energy inflation jointly as risk factors. The t-statistics are in the parentheses. The first two columns report the mean and standard deviation of returns in each row. All returns and inflation variables are annualized and span 1963 to 2019 at the quarterly frequency.

Table 4: Price of Inflation Risks

	A. 8 Average Portfolios		B. 38 Portfolios	
headline λ	0.14		-0.08	
t-stat	(0.47)		(-0.32)	
core λ		-1.03		-1.07
t-stat		(-2.94)		(-3.72)
energy λ		3.86		3.81
t-stat		(1.35)		(1.36)
R^2	0.44	0.98	0.41	0.82

Notes: This table reports the price of risk estimated from the test portfolios. Panel A uses the 8 average portfolios from each asset class as test portfolios. Panel B uses the 38 test portfolios as test assets. In each panel, the first column reports the price of headline inflation and the second column reports the price of core and energy inflation. Price of risk is estimated using two-step procedure and the t-statistics are calculated using Shanken-adjusted standard errors. The second-step R^2 is also reported in the last row.

Table 5: Price of Inflation Risks

	Stock	Treasury	Agen	Corp	Curr	Comm	REIT	Intl	Aver	All
					A. Price of risk					
core λ	-1.26	-0.89	-0.68	-1.09	-0.99	-0.80	-1.06	-0.97	-1.03	-1.07
t-stat	(-2.51)	(-2.43)	(-1.57)	(-2.75)	(-1.96)	(-0.75)	(-2.70)	(-1.69)	(-2.94)	(-3.72)
energy λ	2.02	0.56	-8.25	7.65	2.37	4.18	3.27	8.08	3.86	3.81
t-stat	(0.50)	(0.14)	(-1.06)	(2.01)	(0.26)	(1.41)	(0.41)	(1.31)	(1.35)	(1.36)
R^2	0.26	0.93	0.96	0.75	0.63	0.89	0.23	0.49	0.98	0.82
					B. Weak Identification Test					
p-value	0.101	0.000	0.048	0.000	0.000	0.237	0.210	0.352	0.000	0.000

Notes: Panel A reports the price of risks in various specifications. Columns 1 to 8 use a cross-section of expanded portfolios from each asset class to estimate the price of core and energy risk. Column “Average” uses 8 average portfolios and column “All” uses 38 test portfolios. The price of risk is estimated using two-step procedure and the t-statistics are calculated using Shanken-adjusted standard errors. The second-step R^2 is also reported in the last row. Panel B reports the p-value of weak identification test proposed by Kleibergen and Zhan (2020) for different test portfolios. The null hypothesis is the presence of weak identification.

Table 6: Mimicking Portfolios: Fama-MacBeth Portfolios

	Stock	Treasury	Agen	Corp	Curr	Comm	REIT	Intl	Aver	All
					A. Core					
mean	-1.26	-0.86	-0.68	-1.05	-1.13	-1.38	-1.05	-0.97	-0.91	-0.99
t-stat	(-3.31)	(-2.84)	(-2.09)	(-3.06)	(-3.92)	(-1.16)	(-3.25)	(-2.09)	(-2.92)	(-3.61)
Sharpe ratio	-0.44	-0.36	-0.27	-0.49	-0.64	-0.17	-0.51	-0.31	-0.40	-0.49
					B. Energy					
mean	2.02	0.64	-8.25	6.66	1.34	12.73	3.47	8.08	5.23	5.71
t-stat	(0.61)	(0.19)	(-1.30)	(2.07)	(0.18)	(1.88)	(0.55)	(1.58)	(2.03)	(2.10)
Sharpe ratio	0.09	0.03	-0.18	0.30	0.03	0.36	0.09	0.24	0.28	0.29
					C. Headline					
mean	-2.81	-0.80	-1.39	-1.40	0.79	1.07	0.89	-2.92	0.13	-0.11
t-stat	(-3.36)	(-2.24)	(-3.07)	(-2.85)	(0.88)	(1.61)	(1.12)	(-2.34)	(0.42)	(-0.35)
Sharpe ratio	-0.45	-0.30	-0.46	-0.42	0.17	0.29	0.18	-0.34	0.06	-0.05

Notes: The table reports the characteristics of Fama-MacBeth factor mimicking portfolios for core, energy, and headline inflation. Panels A and B use core and energy inflation as two risk factors. Panel C uses headline inflation as the single risk factor. The table reports the mean, the *t*-statistics, and the Sharpe ratio of these portfolios. The columns indicate the test assets used to construct the mimicking portfolios.

Table 7: Inflation Hedging Properties of Conventional Real Assets

			A. Headline		B.Core and energy			
	Mean	S.D.	headline β	t-stat	core β	t-stat	energy β	t-stat
Currency								
Value-1	-0.01	9.82	1.65	(2.32)	-2.12	(-0.96)	0.21	(2.94)
Value-2	1.16	9.52	1.48	(2.15)	-2.53	(-1.19)	0.20	(2.85)
Value-3	2.52	9.51	1.54	(2.23)	-1.74	(-0.82)	0.20	(2.84)
Value-4	4.14	8.89	1.43	(2.22)	-2.73	(-1.38)	0.21	(3.24)
Dollar-beta-1	0.83	3.80	-0.37	(-1.24)	-0.04	(-0.04)	-0.04	(-1.39)
Dollar-beta-2	1.68	5.61	-0.82	(-1.90)	-1.46	(-1.04)	-0.05	(-1.20)
Dollar-beta-3	2.57	6.93	-0.30	(-0.56)	-1.77	(-1.01)	0.02	(0.34)
Dollar-beta-4	3.65	8.16	0.57	(0.90)	-3.27	(-1.61)	0.12	(1.99)
Dollar-beta-5	3.13	10.03	-0.79	(-1.02)	-3.85	(-1.52)	0.01	(0.07)
Dollar-beta-6	4.87	10.59	-0.62	(-0.75)	-5.05	(-1.91)	0.04	(0.46)
Commodity								
Gold	1.98	17.28	2.14	(1.97)	1.74	(0.91)	0.24	(1.92)
Silver	3.52	31.82	4.95	(2.63)	-0.09	(-0.03)	0.68	(3.06)
Platinum	4.36	20.46	3.40	(2.29)	7.51	(1.63)	0.26	(1.69)
REIT								
Diversified	7.80	20.89	-0.20	(-0.14)	-7.25	(-3.02)	0.27	(1.82)
Healthcare	11.63	19.18	-0.07	(-0.05)	-7.18	(-1.63)	0.09	(0.61)
Industrial/office	6.84	22.08	2.08	(1.44)	-4.90	(-1.93)	0.43	(2.73)
Lodging/resorts	3.62	32.08	1.20	(0.57)	-5.26	(-1.40)	0.38	(1.61)
Residential	9.91	19.65	-0.65	(-0.51)	-9.63	(-4.40)	0.30	(2.18)
Retail	9.21	19.58	1.54	(1.20)	-4.72	(-2.09)	0.38	(2.68)
Self-storage	10.97	20.67	-0.08	(-0.05)	-7.19	(-1.57)	0.06	(0.36)
Unclassified	7.36	19.05	0.55	(0.44)	-5.86	(-2.68)	0.31	(2.24)
TIPS								
Index	1.77	9.53	0.64	(1.61)	4.54	(2.92)	0.01	(0.36)

Notes: This table reports the regression results of the specification $r^e_{i,t} = \alpha_i + \beta^i_\pi \varepsilon_{\pi,t} + u_{i,t}$ for test assets of currencies, commodity futures, REITs, and TIPS. $r^e_{i,t}$ is the return of asset i in excess of the risk-free rate in the US. $\varepsilon_{\pi,t}$ is the shock to respective inflation extracted from the VAR system. Panel A uses headline inflation shock as the risk factor. Panel B uses core and energy inflation jointly as risk factors. The t-statistics are in the parentheses. The first two columns report the mean and standard deviation of returns in each row. All returns and inflation variables are annualized and span 1963 to 2019 at the quarterly frequency.

Table 8: Subsample Analysis

	A. Headline		B. Core and Energy				C. Test Break p-value		
	headline β	t-stat	core β	t-stat	energy β	t-stat	headline	core	energy
			1963-1999						
Stock	-5.42	(-4.20)	-5.19	(-3.26)	-0.24	(-1.01)			
Treasury	-2.88	(-5.52)	-2.77	(-4.31)	-0.20	(-2.03)			
Agency	-2.49	(-4.31)	-2.50	(-3.56)	-0.12	(-1.20)			
Corporate	-3.23	(-5.76)	-3.08	(-4.44)	-0.22	(-2.22)			
Currency	0.12	(0.08)	0.26	(0.09)	0.06	(0.33)			
Commodity	4.55	(2.80)	0.42	(0.21)	0.65	(2.24)			
REIT	-5.79	(-3.76)	-5.91	(-3.29)	-0.37	(-1.43)			
Intl Stock	-5.92	(-4.60)	-5.12	(-3.24)	-0.46	(-1.97)			
			2000-2019						
Stock	2.96	(2.22)	-6.30	(-1.18)	0.35	(2.63)	0.00	0.84	0.03
Treasury	-2.23	(-4.73)	-0.29	(-0.15)	-0.22	(-4.65)	0.36	0.25	0.83
Agency	-1.05	(-4.38)	-0.27	(-0.27)	-0.10	(-4.11)	0.02	0.18	0.82
Corporate	-0.20	(-0.49)	-0.66	(-0.40)	-0.01	(-0.22)	0.00	0.24	0.04
Currency	1.30	(2.67)	-1.87	(-0.96)	0.15	(3.10)	0.37	0.52	0.55
Commodity	12.44	(8.35)	-0.38	(-0.06)	1.24	(8.44)	0.00	0.90	0.07
REIT	3.47	(2.22)	-10.18	(-1.65)	0.44	(2.88)	0.00	0.47	0.01
Intl Stock	3.28	(2.47)	-6.62	(-1.25)	0.39	(2.95)	0.00	0.78	0.00

D. Price of Risk

	1963-1999		2000-2019	
headline λ	-0.25		0.35	
t-stat	(-0.93)		(0.72)	
core λ		-1.05		-0.78
t-stat		(-3.47)		(-2.02)
energy λ		3.12		0.42
t-stat		(1.39)		(0.07)
R^2	0.36	0.42	0.36	0.50

Notes: This table reports the regression results of the specification $r^e_{i,t} = \alpha_i + \beta_i \varepsilon_{\pi,t} + u_{i,t}$ for 8 average portfolios in each asset class. Panel A uses headline inflation shock as the risk factor. Panel B uses core and energy inflation jointly as risk factors. The t-statistics are in the parentheses. The upper panels show the results from 1963-1999 and the lower panels show the results from 2000-2019. Panel C shows the p-value of tests on equal β over the two sub-sample. Panel D reports the price of risk estimated from 38 test portfolios. Price of risk is estimated using two-step procedure and the t-statistics are calculated using Shanken-adjusted standard errors. The second-step R^2 is also reported in the last row.

Table 9: Price of Inflation Risks with Macroeconomic Factors

	Cons	Cons/Dur	IP	Pay	Unem	HHL	Unf Cons	Cap	Itmdr	CAPM
core	-1.06	-1.04	-1.07	-1.07	-1.06	-1.04	-1.07	-1.08	-1.05	-1.06
t-stat	(-3.69)	(-3.67)	(-3.51)	(-3.27)	(-3.39)	(-3.48)	(-3.70)	(-3.72)	(-3.65)	(-3.39)
energy	3.90	4.38	4.08	3.68	3.84	3.97	3.98	3.94	4.50	3.83
t-stat	(1.29)	(1.36)	(1.38)	(1.33)	(1.36)	(1.29)	(1.44)	(1.38)	(1.58)	(1.35)
macro	0.10	0.17	-0.34	-0.08	0.11	0.46	0.00	-0.31	2.03	6.78
t-stat	(0.18)	(0.32)	(-0.24)	(-0.16)	(0.26)	(0.62)	(0.26)	(-0.59)	(1.76)	(2.93)
macro2		-2.62				-0.01				
t-stat		(-0.67)				(-0.58)				
R^2	0.82	0.82	0.82	0.82	0.82	0.81	0.82	0.80	0.72	0.73

Notes: This table reports the price of risks in various specifications with the inclusion of macroeconomic factors including consumption growth (Cons), durable consumption growth (Cons/Dur), industrial production growth (IP), payroll growth (Pay), unemployment rate growth(Unem), long-run consumption growth rate and short-run consumption growth news (HHL), unfiltered consumption growth (Unf Cons), and capital share growth (Cap), intermediary factor (Itmdr), and the aggregate stock market return (CAPM).

Table 10: Inflation and Real Growth

	headline	t-stat	R^2	core	t-stat	energy	t-stat	R^2
				1 quarter				
GDP	-0.14	(-1.21)	0.02	-0.21	(-1.88)	0.00	(-0.23)	0.03
Consumption	-0.22	(-2.42)	0.08	-0.22	(-2.32)	-0.01	(-0.86)	0.07
Dividend	-0.27	(-1.15)	0.02	-0.67	(-4.27)	0.04	(0.96)	0.06
				1 year				
GDP	-0.75	(-2.34)	0.08	-0.70	(-2.24)	-0.05	(-1.05)	0.07
Consumption	-0.66	(-2.24)	0.09	-0.46	(-1.81)	-0.05	(-1.12)	0.05
Dividend	-1.26	(-1.12)	0.03	-2.93	(-5.78)	0.18	(0.95)	0.11

Notes: This table reports the results of regressions of future real growth on current headline, core, and energy inflation at one-quarter and one-year horizons. The t-statistics are in the parentheses.

Table 11: Cash Flow and Discount Rate News Exposures

	Cash Flow News				Discount Rate News			
	core	*t*-stat	energy	*t*-stat	core	*t*-stat	energy	*t*-stat
Stock	-2.14	(-4.12)	-0.01	(-0.23)	4.23	(3.47)	-0.19	(-2.05)
Consumer	-6.23	(-6.03)	-0.19	(-2.43)	1.00	(1.40)	-0.18	(-3.39)
Manufacturing	-1.80	(-3.97)	0.00	(0.07)	3.16	(2.98)	-0.28	(-3.46)
HiTech	-4.61	(-3.49)	0.06	(0.65)	2.23	(1.46)	-0.12	(-1.04)
Health	-0.86	(-1.67)	-0.16	(-4.19)	6.16	(3.64)	-0.12	(-0.94)
Others	-4.04	(-3.71)	0.03	(0.41)	4.04	(4.13)	-0.08	(-1.07)
BM1 growth	-4.96	(-5.58)	-0.11	(-1.60)	2.57	(2.57)	-0.24	(-3.14)
BM2	-2.44	(-2.83)	0.00	(-0.03)	3.07	(3.55)	-0.10	(-1.55)
BM3	-2.28	(-2.76)	-0.03	(-0.47)	2.73	(3.37)	-0.14	(-2.26)
BM4	0.71	(0.80)	0.12	(1.80)	6.27	(4.33)	-0.12	(-1.07)
BM5 value	1.27	(1.17)	0.08	(0.92)	7.17	(4.35)	-0.14	(-1.09)
World	-3.07	(-4.10)	0.03	(0.53)	3.27	(3.77)	-0.08	(-1.22)
North America	-0.89	(-1.98)	0.01	(0.18)	5.18	(4.35)	-0.14	(-1.62)
Europe	-5.54	(-3.99)	-0.04	(-0.40)	0.69	(1.19)	-0.14	(-3.29)
Far East	-2.82	(-3.06)	0.08	(1.18)	3.03	(1.96)	0.03	(0.30)
REITs Index	-4.63	(-2.58)	0.28	(3.01)	1.93	(1.98)	-0.04	(-0.81)
Equity	-5.68	(-2.95)	0.34	(3.38)	1.41	(1.52)	-0.03	(-0.64)
Mortgage	-4.81	(-1.64)	-0.10	(-0.65)	0.74	(1.79)	-0.01	(-0.41)
Hybrid	-1.16	(-0.64)	0.25	(2.62)	3.50	(2.84)	0.10	(1.51)

Notes: This table reports the regression results of the specification $r^e_{i,t} = \alpha_i + \beta^i_\pi \varepsilon_{\pi,t} + u_{i,t}$ for 8 average portfolios in each asset class. $r^e_{i,t}$ is cash flow or discount rate news of the corresponding stock portfolios. $\varepsilon_{\pi,t}$ is the shock to respective inflation extracted from the VAR system. The *t*-statistics are in the parentheses.

Table 12: Inflation Announcement Event Study

A. Fed Funds Futures

	core	*t*-stat	headline	*t*-stat
(1)	2.05	(3.33)		
(2)			0.50	(1.11)
(3)	2.18	(3.15)	-0.20	(-0.41)

B. Stock futures

	core	*t*-stat	headline	*t*-stat	FFF	*t*-stat
(1)	-1.49	(-6.33)				
(2)			-0.73	(-4.57)		
(3)	-1.25	(-5.02)	-0.44	(-2.68)		
(4)					-0.11	(-3.93)
(5)	-1.32	(-5.37)			-0.08	(-3.08)

Notes: This table reports the regression results of core and headline inflation surprises on changes in Fed Funds futures (Panel A) and S&P 500 futures (Panel B). The inflation surprises are the difference between the median forecast from Bloomberg and the actual announcement. The futures price changes are from 2 minutes before the announcement to 20 minutes after the announcement. In Panel B, some regressions control for the changes in Fed Funds futures (FFF). The *t*-statistics are in the parentheses. The sample is monthly from 1997 to 2019.

Table 13: Hedging Inflation, Expected Inflation, and Inflation Shocks

A. Headline

	headline	*t*-stat	headline expect.	*t*-stat	headline shock	*t*-stat
Stock	-1.35	(-1.96)	-1.37	(-1.38)	-1.33	(-1.38)
Treasury	-1.44	(-5.38)	-0.29	(-0.77)	-2.53	(-7.05)
Agency	-1.02	(-4.47)	-0.30	(-0.92)	-1.62	(-5.44)
Corporate	-1.18	(-4.58)	-0.75	(-2.04)	-1.59	(-4.38)
Currency	0.89	(1.74)	0.00	(0.00)	1.04	(1.95)
Commodity	4.50	(5.20)	0.20	(0.17)	8.59	(7.51)
REIT	-0.04	(-0.05)	-0.50	(-0.36)	0.25	(0.21)
IntlStock	-1.19	(-1.72)	-1.19	(-1.19)	-1.19	(-1.23)

B.Core and Energy

	core	*t*-stat	energy	*t*-stat	core expect.	*t*-stat	core shock	*t*-stat	energy shock	*t*-stat
Stock	-2.43	(-2.87)	0.16	(1.36)	-0.91	(-0.93)	-5.60	(-3.69)	0.21	(1.81)
Treasury	-0.68	(-2.03)	-0.20	(-4.48)	-0.20	(-0.54)	-2.51	(-4.26)	-0.20	(-4.56)
Agency	-0.69	(-2.35)	-0.10	(-2.93)	-0.27	(-0.86)	-2.28	(-4.32)	-0.09	(-2.74)
Corporate	-1.16	(-3.52)	-0.06	(-1.41)	-0.58	(-1.58)	-2.99	(-4.95)	-0.05	(-1.06)
Currency	-0.19	(-0.19)	0.12	(2.29)	-0.05	(-0.05)	-1.08	(-0.61)	0.13	(2.52)
Commodity	-1.04	(-1.03)	1.08	(8.08)	0.01	(0.01)	-0.07	(-0.04)	1.10	(8.19)
REIT	-2.49	(-2.05)	0.26	(2.11)	-0.75	(-0.57)	-6.76	(-3.34)	0.30	(2.45)
IntlStock	-2.38	(-2.76)	0.14	(1.27)	-0.87	(-0.90)	-5.81	(-3.75)	0.19	(1.71)

Notes: This table reports the regression results of the specification $r^e_{i,t} = \alpha_i + \beta^i_\pi x_{\pi,t} + u_{i,t}$ for 8 average portfolios in each asset class. $r^e_{i,t}$ is the return of asset i in excess of the risk-free rate in the US. $x_{\pi,t}$ is the inflation level, expected inflation, and inflation shock for headline (Panel A) and core and energy (Panel B). The *t*-statistics are in the parentheses.All returns and inflation variables are annualized and span 1963 to 2019 at the quarterly frequency.

Table 14: Hedging Inflation, Expected Inflation, and Inflation Shocks over Longer Horizons

A. Headline

	headline	*t*-stat	headline expect.	*t*-stat	headline shock	*t*-stat
Stock	-0.18	(-1.72)	-0.95	(-1.06)	-1.94	(-1.52)
Treasury	-0.13	(-5.18)	-0.53	(-1.37)	-1.64	(-3.19)
Agency	-0.12	(-4.39)	-0.20	(-0.85)	-1.92	(-7.22)
Corporate	-0.15	(-4.25)	-0.49	(-1.27)	-2.00	(-4.65)
Currency	0.23	(2.59)	2.01	(2.65)	1.39	(1.03)
Commodity	0.33	(1.71)	0.24	(0.13)	5.15	(2.95)
REIT	-0.12	(-0.66)	-1.51	(-0.93)	1.11	(0.46)
IntlStock	-0.17	(-1.87)	-0.56	(-0.70)	-2.19	(-1.31)

B.Core and Energy

	core	*t*-stat	energy	*t*-stat	core expect.	*t*-stat	core shock	*t*-stat	energy shock	*t*-stat
Stock	-0.15	(-1.84)	0.00	(-0.13)	-0.44	(-0.44)	-4.14	(-3.12)	0.40	(1.41)
Treasury	-0.05	(-1.35)	-0.04	(-3.10)	-0.38	(-1.06)	-1.41	(-2.11)	-0.21	(-3.15)
Agency	-0.04	(-1.37)	-0.03	(-3.46)	-0.13	(-0.76)	-2.11	(-7.48)	-0.11	(-2.64)
Corporate	-0.08	(-1.59)	-0.03	(-2.31)	-0.30	(-0.89)	-2.57	(-4.45)	-0.04	(-0.44)
Currency	0.22	(1.64)	0.02	(0.82)	1.48	(1.66)	-0.92	(-0.37)	0.26	(2.27)
Commodity	-0.24	(-1.51)	0.21	(5.16)	0.42	(0.30)	-4.68	(-3.21)	2.00	(6.23)
REIT	-0.21	(-1.14)	0.06	(1.38)	-1.24	(-0.78)	-3.01	(-1.19)	0.73	(1.72)
IntlStock	-0.11	(-1.02)	-0.02	(-0.38)	-0.14	(-0.18)	-4.66	(-3.63)	0.34	(0.93)

Notes: This table reports the regression results of the specification $r^e_{i,t} = \alpha_i + \beta^i_\pi x_{\pi,t} + u_{i,t}$ for 8 average portfolios in each asset class over the long horizon of 8 quarters. $r^e_{i,t}$ is the return of asset i in excess of the risk-free rate in the US. $x_{\pi,t}$ is the inflation level, expected inflation, and inflation shock for headline (Panel A) and core and energy (Panel B). The *t*-statistics reported in the parentheses have adjusted for the time-series autocorrelation. All returns and inflation variables are annualized and span 1963 to 2019 at the quarterly frequency.

Table 15: Flexible vs. Sticky Inflation

A. Asset Return Exposures

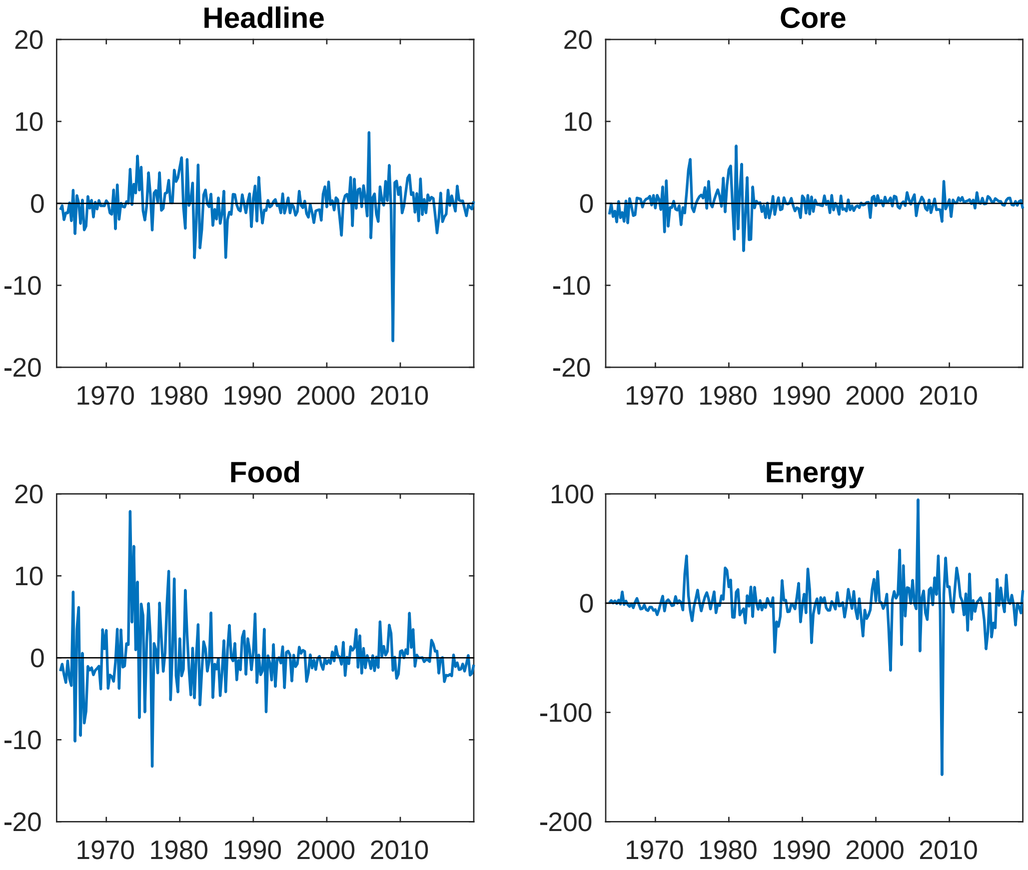
	sticky	*t*-stat	flexible	*t*-stat
Stock	-4.68	(-2.99)	0.25	(0.61)
Treasury	-1.12	(-1.86)	-0.93	(-5.93)
Agency	-0.94	(-1.93)	-0.51	(-4.20)
Corporate	-1.61	(-2.70)	-0.39	(-2.56)
Currency	-1.14	(-0.69)	0.41	(2.16)
Commodity	-1.53	(-0.87)	3.88	(8.51)
REIT	-4.35	(-2.38)	0.61	(1.38)
Intl Stock	-4.95	(-3.27)	0.23	(0.58)

B. Price of Risk

	8 Average Portfolios	38 Portfolios
sticky λ	-1.50	-1.45
t-stat	(-2.61)	(-3.49)
flexible λ	0.45	-0.21
t-stat	(0.47)	(-0.24)
R^2	0.89	0.78

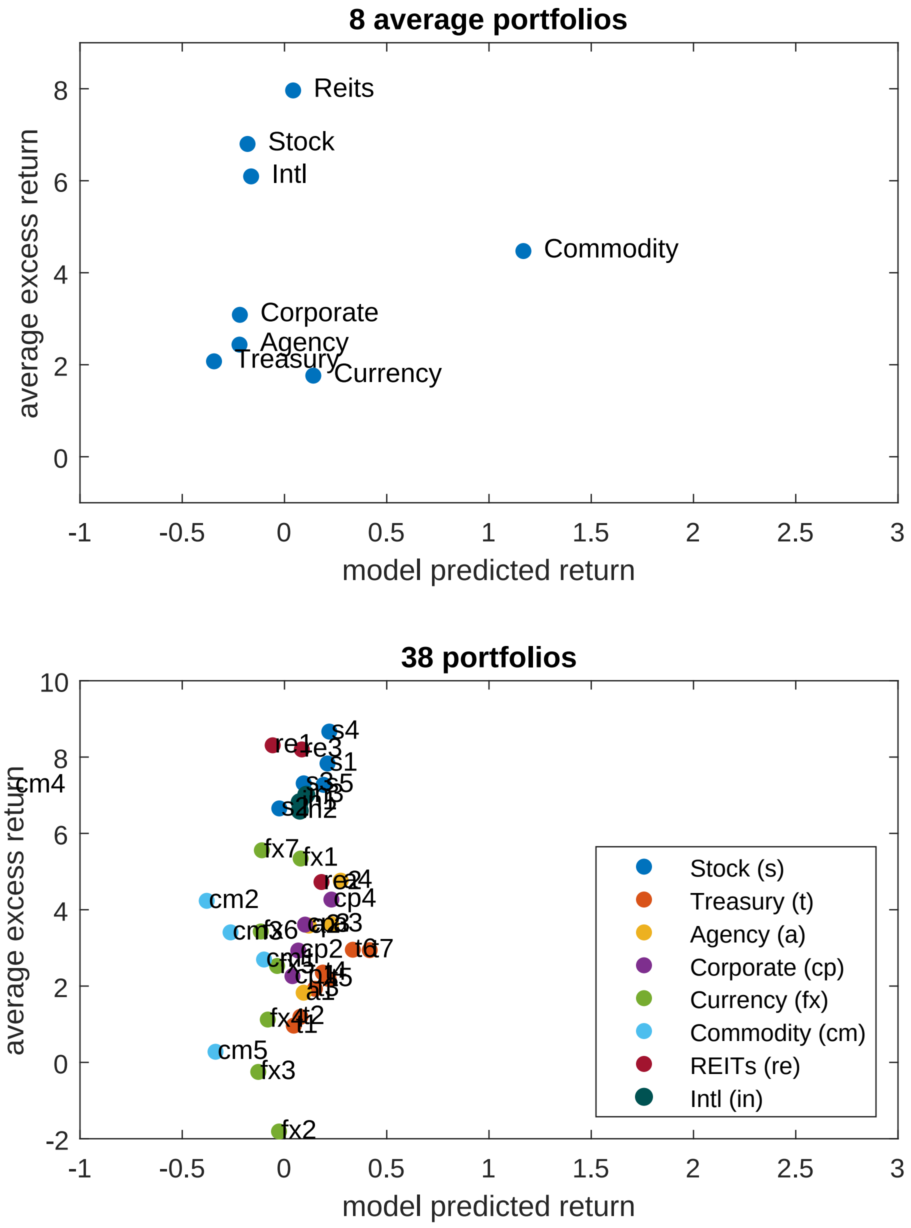
Notes: This table reports the two-step asset pricing results for flexible and sticky inflation. Panel A reports the asset return exposures for the 8 average portfolios, and Panel B reports the price of risk estimates for both 8 portfolios and 38 portfolios. The t-statistics are in the parentheses.

Figure 1: Inflation



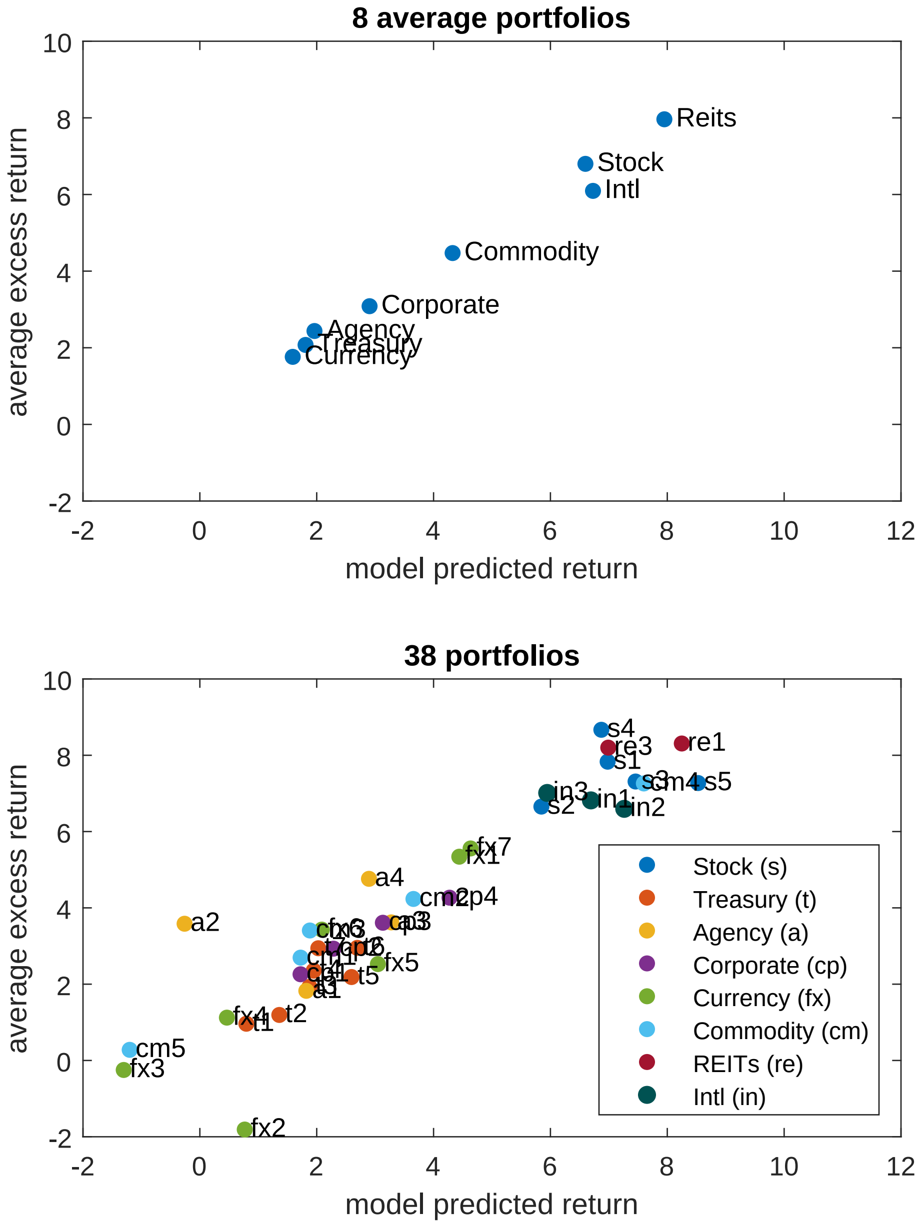
Notes: This figure plots the time-series of headline, core, food, and energy inflation shocks extracted from VAR described in the main text. Data are quarterly from 1963Q3 to 2019Q4.

Figure 2: Average Returns vs. Model Predicted Returns: Headline Inflation



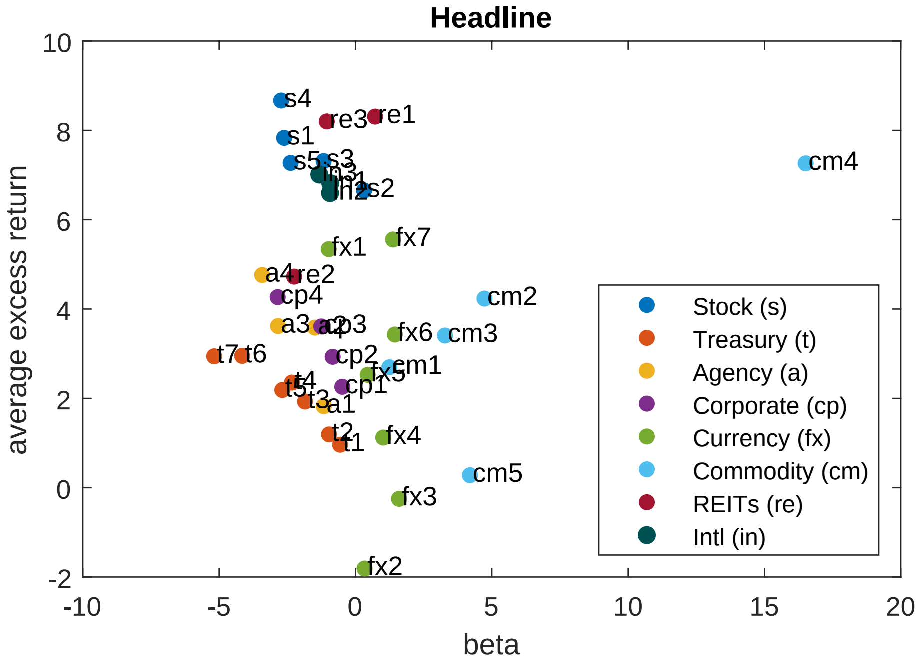
Notes: This figure plots the average excess return of the 8 average portfolios and 38 portfolios against their model predicted returns with headline inflation only as the risk factor. The horizontal axis are model predicted returns, and the vertical axis shows the average excess returns. Each dot represents a portfolio. Abbreviation correspondence: s1 cons, s2 manu, s3 tech, s4 health, s5 others; t1-t7, a1-a4, cp1-cp4, from short to long maturity; fx1 dollar carry, fx2-fx7, from low interest rate to high interest rate; cm1 livestock, cm2 ind metal, cm3 pre metal, cm4 energy, cm5 agriculture; re1 equity, re2 mortgage, re3 hybrid, in1 North America, in2 Europe, in3 Far East.

Figure 3: Average Returns vs. Model Predicted Returns: Core and Energy Inflation



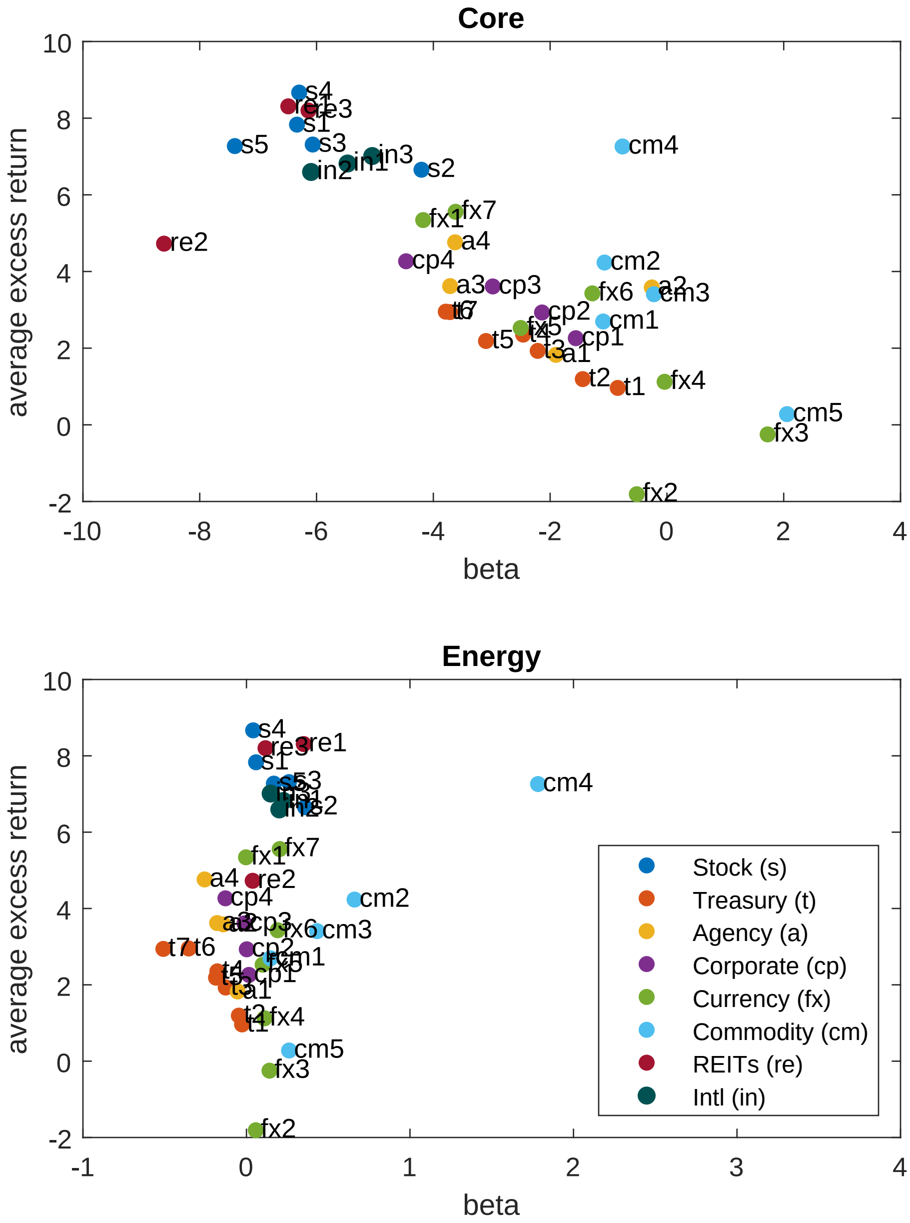
Notes: This figure plots the average excess return of the 8 average portfolios and 38 portfolios against their model predicted returns with core and energy inflation risk factors. The horizontal axis is model predicted returns, and the vertical axis shows the average excess returns. Each dot represents a portfolio. Abbreviation correspondence: s1 cons, s2 manu, s3 tech, s4 health, s5 others; t1-t7, a1-a4, cp1-cp4, from short to long maturity; fx1 dollar carry, fx2-fx7, from low interest rate to high interest rate; cm1 livestock, cm2 ind metal, cm3 pre metal, cm4 energy, cm5 agriculture; re1 equity, re2 mortgage, re3 hybrid, in1 North America, in2 Europe, in3 Far East.

Figure 4: Headline Inflation Betas and Average Returns



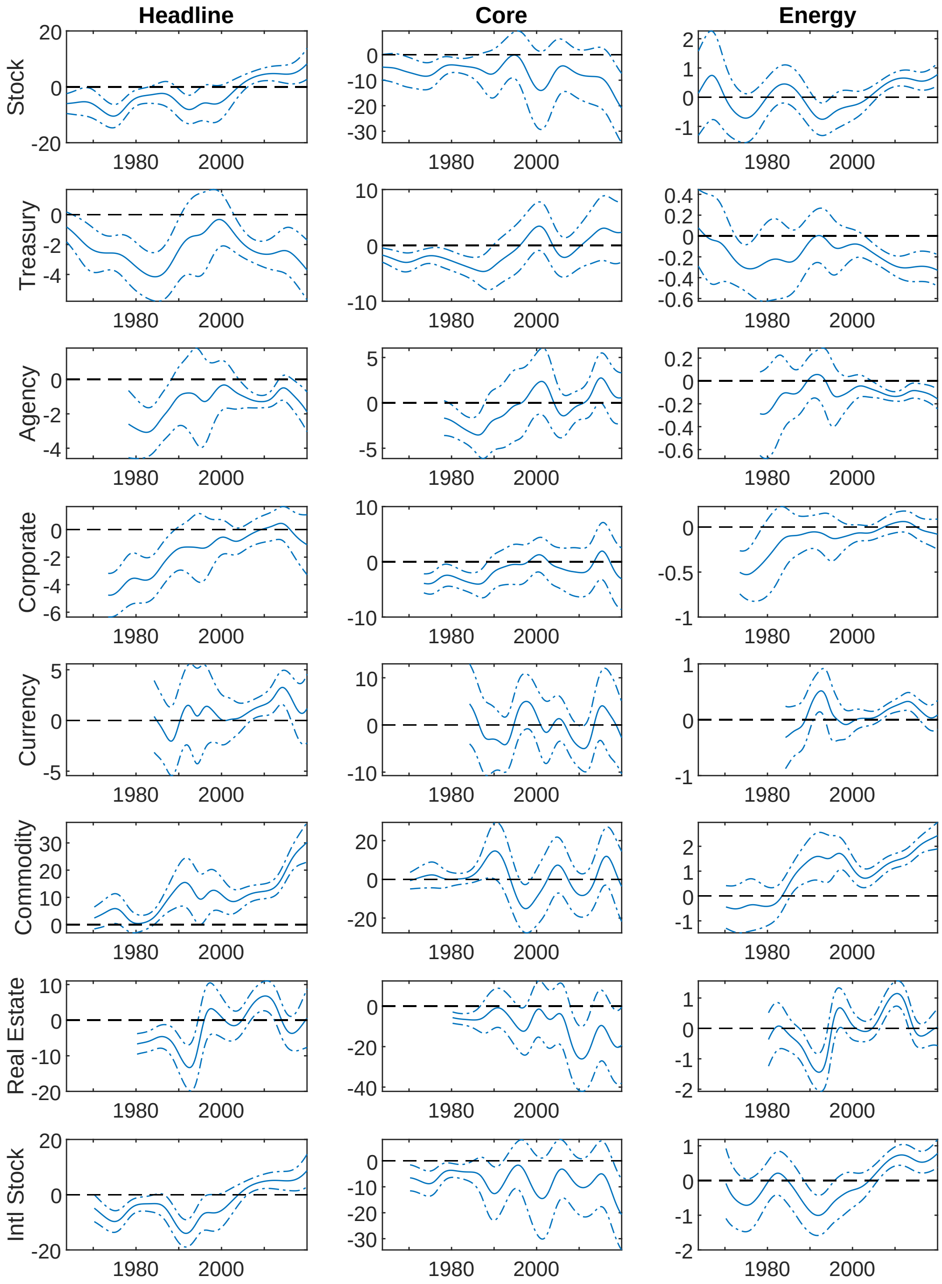
Notes: This figure plots the average excess return of the 38 test portfolios against their headline inflation betas. The horizontal axis shows headline inflation betas and the vertical axis shows the average excess returns. Each dot represents a test portfolio, and different colors refer to assets from different asset classes. Abbreviation correspondence: s1 cons, s2 manu, s3 tech, s4 health, s5 others; t1-t7, a1-a4, cp1-cp4, from short to long maturity; fx1 dollar carry, fx2-fx7, from low interest rate to high interest rate; cm1 livestock, cm2 ind metal, cm3 pre metal, cm4 energy, cm5 agriculture; re1 equity, re2 mortgage, re3 hybrid, in1 North America, in2 Europe, in3 Far East.

Figure 5: Core and Energy Inflation Betas and Average Returns



Notes: This figure plots the average excess return of the 38 test portfolios against their core and energy inflation betas (bivariate). The horizontal axis shows core and energy inflation betas and the vertical axis shows the average excess returns. Each dot represents a test portfolio, and different colors refer to assets from different asset classes. Abbreviation correspondence: s1 cons, s2 manu, s3 tech, s4 health, s5 others; t1-t7, a1-a4, cp1-cp4, from short to long maturity; fx1 dollar carry, fx2-fx7, from low interest rate to high interest rate; cm1 livestock, cm2 ind metal, cm3 pre metal, cm4 energy, cm5 agriculture; re1 equity, re2 mortgage, re3 hybrid, in1 North America, in2 Europe, in3 Far East.

Figure 6: Inflation Beta and Average Returns



Notes: This figure plots the time-varying estimates of betas. The core and energy betas are in a bi-variate regression. The estimates are from a Gaussian kernel estimator with bandwidth of 0.05. The 90% confidence intervals are plotted.

Appendix

A Model Details

In this section of the appendix, we list a detailed description of the model, including the optimization of different economic agents, optimality conditions, and the intermediate steps of solving for asset prices.

A.1 Households

The households' optimization problem is stated in the main text, which leads to the following Euler equation:

$$E_t M^{\$}_{t+1} R_{t+1} = 1, \tag{15}$$

where $M^{\$}_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \frac{P_{h,t} P_t}{P_{h,t+1} P_{t+1}}$.

Particularly, the Euler equation for nominal risk-free rate characterizes the aggregate-demand equilibrium relation in this economy, i.e.:

$$E_t M^{\$}_{t+1}(1 + i_t) = 1. \tag{16}$$

Plug in $M^{\$}_{t+1}$ and the Taylor rule, the Euler equation can be rewritten as:

$$-\eta_c E_t(c_{c.t+1} - c_{c,t}) - \kappa_e[E_t(c_{e,t+1} - c_{e,t}) + E_t(\delta_{t+1} - \delta_t)] - E_t \pi_{t+1} + \phi_\pi \pi_t = 0, \tag{17}$$

where $\eta_c = \gamma \hat{\alpha}_c + \frac{1-\hat{\alpha}_c}{\phi} > 0, \kappa_e = (\gamma - \frac{1}{\phi})(1 - \hat{\alpha}_c) < 0$.

A.2 Product

The firm's optimal price setting decision is described in the following optimization problem:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t M^{\$}_{t,t+k}[P_t^* Y_{t+k|t} - \Psi(Y_{t+k|t})] \tag{18}$$

$$s.t. : Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon_{t+k}} C_{c,t+k} \tag{19}$$

where $Y_{t+k|t}$ is the production if the firm still faces a price P_t^* in period $t+k$, $M^{\$}_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\gamma} \frac{P_{h,t}}{P_{h,t+k}} \frac{P_t}{P_{t+k}}$ is the nominal stochastic discount factor from period t to $t+k$, and $\Psi(Y_{t+k|t})$

is the total cost of producing $Y_{t+k|t}$ units of core good. The firm's optimality condition is:

$$\sum_{k=0}^{\infty} \theta^k E_t M^{\$}_{t,t+k} \left[Y_{t+k|t} + (P_t^* - \Psi'(Y_{t+k|t})) \frac{\partial Y_{t+k|t}}{\partial P_t^*} \right] = 0 \tag{20}$$

After log-linearization, we obtain the standard New-Keynesian Phillips Curve:

$$\pi_t = \beta E_t \pi_{t+1} - \lambda(-mc_t - \mu_t), \tag{21}$$

where $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\bar{\varepsilon}}$ and mc_t is the real marginal cost of core production, or the inverse of markup, which can be expressed as:

$$mc_t = \kappa_c c_{c,t} + \kappa_e c_{e,t} + \kappa_\delta \delta_t, \tag{22}$$

where $\kappa_c = \frac{\varphi+\alpha}{1-\alpha} + \gamma\hat{\alpha}_c + \frac{1-\hat{\alpha}_c}{\phi} > 0$.

Core inflation depends on future expected core inflation and the deviation of markup from the desired level. λ captures the relative weight of future expectation and current markup deviation. The marginal cost increases with the production of core goods ($\kappa_c > 0$) both because the production technology is decreasing return to scale and the increase in wage when core consumption is higher. A positive energy demand or supply shock reduces marginal cost of production ($\kappa_e < 0$) because it raises the marginal utility of consumption of core goods, which leads to a lower wage level.

Equation (17) and (21) define the equilibrium of the economy.

A.3 Asset Prices

We use the Euler equation of the model to price core stocks, nominal bonds, foreign currencies, and commodity futures.

A.3.1 Core Stock Returns

The dividend is equal to core output net of labor cost, and we express the dividend in the unit of headline price.

$$D_t = \frac{1}{P_{h,t}}(C_{c,t} - \frac{W_t}{P_t} N_t).$$

At the steady state, labor income accounts for $(1-\alpha)\exp(-\bar{\mu})$ share of the total output, so that dividend accounts for the remaining $1-(1-\alpha)\exp(-\bar{\mu})$ share. Log-linearize this equation around the steady state:

$$d_t = \frac{1}{1-(1-\alpha)\exp(-\bar{\mu})}[c_{c,t} - (1-\alpha)\exp(-\bar{\mu})(w_t - p_t + n_t)] - p_{h,t}.$$

To calculate the return to core stocks, we conduct a Campbell-Shiller decomposition:

$$r_{s,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta d_{t+1},$$

where z_t is the price-dividend ratio of the core stock and $z_t = z_\mu \mu_t + z_e c_{e,t} + z_\delta \delta_t$. The coefficients z_μ, z_e, z_δ can be solved through the Euler equation. Return loadings r_μ, r_e, r_δ follow straightforwardly.

A.3.2 Nominal Bond Returns

Consider a one-period and a two-period default-free nominal bond. Denote $P_t^{(1)}$ the price of one-period bond at time t, and $P_t^{(2)}$the price of two-period bond at time t. The following Euler equation prices the nominal bond.

$$E_t M_{t+1}^{\$} P_{t+1}^{(1)} = P_t^{(2)}, E_t M_{t+1}^{\$} = P_t^{(1)}.$$

A.3.3 Currency Returns

The pricing of foreign currencies follow $\Delta q_{t+1} = m_{t+1}^* - m_{t+1}$,as in the main text.

A.3.4 Commodity Futures

Denote F_t the price of commodity future. The following Euler equation prices the commodity future.

$$E_t M_{t+1}^{\$} P_{e,t+1} P_{t+1} = E_t M_{t+1}^{\$} F_t.$$

B Proofs

B.1 Proof of Proposition 1

The model has three exogenous state variables and no endogenous state variable. After log-linearization, all variables can be written as linear functions of the three state variables. So we can postulate:

$$c_{c,t} = c_{c,\mu}\mu_t + c_{c,e}c_{e,t} + c_{c,\delta}\delta_t, \pi_t = \pi_\mu\mu_t + \pi_e c_{e,t} + \pi_\delta\delta_t.$$

We plug in the postulated solution of core output and inflation into the equations (21) and (17):

$$\pi_\mu\mu_t+\pi_e c_{e,t}+\pi_\delta\delta_t = \beta(\rho_\mu\pi_\mu\mu_t+\rho_e\pi_e c_{e,t}+\rho_\delta\pi_\delta\delta_t)+\lambda[\kappa_c(c_{c,\mu}\mu_t+c_{c,e}c_{e,t}+c_{c,\delta}\delta_t)+\kappa_e c_{e,t}+\kappa_\delta\delta_t]+\lambda\mu_t$$

$$-\eta_c[(\rho_\mu-1)c_{c,\mu}\mu_t+(\rho_e-1)c_{c,e}c_{e,t}+(\rho_\delta-1)c_{c,\delta}\delta_t]-\kappa_e[(\rho_e-1)c_{e,t}+(\rho_\delta-1)\delta_t]$$

$$-(\rho_\mu-\phi_\pi)\pi_\mu\mu_t-(\rho_e-\phi_\pi)\pi_e c_{e,t}-(\rho_\delta-\phi_\pi)\pi_\delta\delta_t = 0$$

These two equations need to hold for all values of μ_t, so that:

$$(1-\beta\rho_\mu)\pi_\mu - \lambda\kappa_c c_{c,\mu} - \lambda = 0,$$

$$\eta_c(1-\rho_\mu)c_{c,\mu} + (\phi_\pi-\rho_\mu)\pi_\mu = 0.$$

We can solve for $c_{c,\mu}$ and π_μ from the two equations:

$$c_{c,\mu} = \frac{-\lambda(\phi_\pi-\rho_\mu)}{\eta_c(1-\rho_\mu)(1-\beta\rho_\mu)+\lambda\kappa_c(\phi_\pi-\rho_\mu)},$$

$$\pi_\mu = -\frac{\eta_c(1-\rho_\mu)}{\phi_\pi-\rho_\mu}c_{c,\mu} = \frac{\lambda\eta_c(1-\rho_\mu)}{\eta_c(1-\rho_\mu)(1-\beta\rho_\mu)+(\phi_\pi-\rho_\mu)\lambda\kappa_c}.$$

Since $\phi_\pi > 1, \rho_\mu < 1$, and recall that $\eta_c > 0, \lambda > 0, \kappa_c > 0$,we can easily see $c_{c,\mu} < 0, \pi_\mu > 0$.

Similarly, we can derive the conditions under which the two equations hold for all values of energy supply shock, $c_{e,t}$.

$$(1-\beta\rho_e)\pi_e - \lambda\kappa_c c_{c,e} - \lambda\kappa_e = 0,$$

$$\eta_c(1-\rho_e)c_{c,e} + \kappa_e(1-\rho_e) + (\phi_\pi-\rho_e)\pi_e = 0.$$

We can solve for $c_{c,e}$ and π_e from the two equations:

$$c_{c,e} = \frac{-\kappa_e\left[1-\rho_e+\frac{\lambda(\phi_\pi-\rho_e)}{1-\beta\rho_e}\right]}{\eta_c(1-\rho_e)+\frac{\lambda\kappa_c(\phi_\pi-\rho_e)}{1-\beta\rho_e}}$$

$$\pi_e = \frac{\lambda}{1-\beta\rho_e}(\kappa_e+\kappa_c c_{c,e}) = \frac{\lambda}{1-\beta\rho_e}\frac{\kappa_e(\eta_c-\kappa_c)(1-\rho_e)}{\eta_c(1-\rho_e)+\frac{\lambda\kappa_c(\phi_\pi-\rho_e)}{1-\beta\rho_e}} = -\frac{\lambda\kappa_e(1-\rho_e)\frac{\varphi+\alpha}{1-\alpha}}{(1-\beta\rho_e)\left[\eta_c(1-\rho_e)+\frac{\lambda\kappa_c(\phi_\pi-\rho_e)}{1-\beta\rho_e}\right]}$$

Since $\kappa_e < 0$, it is straightforward to see that $c_{c,e} > 0, \pi_e > 0$.

For the energy demand shock δ_t, we can similarly solve for $c_{c,\delta}$ and π_δ. They have almost the same expression as the energy supply shock except for potential different persistence. We skip the derivation here.

B.2 Proof of Proposition 2

The real stochastic discount factor in comsumption basket is equal to $M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$, where C_t is the consumption basket. After log-linearization, we have:

$$m_{t+1} = -\gamma(c_{t+1}-c_t) = -\gamma\left[\hat{\alpha}_c(c_{c,t+1}-c_{c,t}) + (1-\hat{\alpha}_c)(c_{e,t+1}-c_{e,t}+\delta_{t+1}-\delta_t)\right]$$

$$= -\gamma[\hat{\alpha}_c c_{c,\mu}(\rho_\mu-1)\mu_t + (\hat{\alpha}_c c_{c,e}+1-\hat{\alpha}_c)(\rho_e-1)c_{e,t} + (\hat{\alpha}_c c_{c,\delta}+1-\hat{\alpha}_c)(\rho_\delta-1)\delta_t]$$

$$+(-\gamma\hat{\alpha}_c c_{c,\mu})\sigma_\mu\varepsilon_{\mu,t+1} + [-\gamma(\hat{\alpha}_c c_{c,e}+1-\hat{\alpha}_c)]\sigma_e\varepsilon_{e,t+1} + [-\gamma(\hat{\alpha}_c c_{c,\delta}+1-\hat{\alpha}_c)]\sigma_\delta\varepsilon_{\delta,t+1}$$

If we write the SDF as $m_{t+1} = m_\mu(1-\rho_\mu)\mu_t + m_e(1-\rho_e)c_{e,t} + m_\delta(1-\rho_\delta)\delta_t - \lambda_\mu\sigma_\mu\varepsilon_{\mu,t+1} - \lambda_e\sigma_e\varepsilon_{e,t+1} - \lambda_\delta\sigma_\delta\varepsilon_{\delta,+1}$, we can obtain:

$$m_\mu = \gamma\hat{\alpha}_c c_{c,\mu} < 0, m_e = \gamma(\hat{\alpha}_c c_{c,e}+1-\hat{\alpha}_c) > 0,$$

$$m_\delta = \gamma(\hat{\alpha}_c c_{c,e}+1-\hat{\alpha}_c) > 0,$$

$$\lambda_\mu = \gamma\hat{\alpha}_c c_{c,\mu} < 0, \lambda_e = \gamma(\hat{\alpha}_c c_{c,e}+1-\hat{\alpha}_c) > 0, \lambda_\delta = \gamma(\hat{\alpha}_c c_{c,\delta}+1-\hat{\alpha}_c) > 0.$$

All signs directly follow Proposition 1.

B.3 Proof of Proposition 3

Log dividend is written as:

$$d_t = \frac{1}{1-(1-\alpha)\exp(-\bar{\mu})}[c_{c,t} - (1-\alpha)\exp(-\bar{\mu})(w_t - p_t + n_t)] - p_{h,t}$$

$$= \frac{1}{1-(1-\alpha)\exp(-\bar{\mu})}[c_{c,t} - (1-\alpha)\exp(-\bar{\mu})(\gamma[(\hat{\alpha}_c c_{c,t} + (1-\hat{\alpha}_c)(c_{e,t}+\delta_t)] + \frac{\varphi+1}{1-\alpha}c_{c,t})] - \frac{1-\hat{\alpha}_c}{\phi}(c_{c,t} - c_{e,t} - \delta_t).$$

Denote $d_t = d_\mu \mu_t + d_e c_{e,t} + d_\delta \delta_t$,we have:

$$d_\mu = \left[\frac{1-(1-\alpha)\exp(-\bar{\mu})(\gamma\hat{\alpha}_c + \frac{\varphi+1}{1-\alpha})}{1-(1-\alpha)\exp(-\bar{\mu})} - \frac{1-\hat{\alpha}_c}{\phi}\right] c_{c,\mu},$$

$$d_e = \left[\frac{1-(1-\alpha)\exp(-\bar{\mu})(\gamma\hat{\alpha}_c + \frac{\varphi+1}{1-\alpha})}{1-(1-\alpha)\exp(-\bar{\mu})} - \frac{1-\hat{\alpha}_c}{\phi}\right] c_{c,e} - \frac{(1-\alpha)\exp(-\bar{\mu})\gamma(1-\hat{\alpha}_c)}{1-(1-\alpha)\exp(-\bar{\mu})} + \frac{1-\hat{\alpha}_c}{\phi},$$

$$d_\delta = \left[\frac{1-(1-\alpha)\exp(-\bar{\mu})(\gamma\hat{\alpha}_c + \frac{\varphi+1}{1-\alpha})}{1-(1-\alpha)\exp(-\bar{\mu})} - \frac{1-\hat{\alpha}_c}{\phi}\right] c_{c,\delta} - \frac{(1-\alpha)\exp(-\bar{\mu})\gamma(1-\hat{\alpha}_c)}{1-(1-\alpha)\exp(-\bar{\mu})} + \frac{1-\hat{\alpha}_c}{\phi}.$$

When $\exp(-\bar{\mu}) < \frac{1}{(1-\alpha)(\gamma+\frac{\varphi+1}{1-\alpha})}$, and $\hat{\alpha}_c$ is close to 1, $1-(1-\alpha)\exp(-\bar{\mu})(\gamma+\frac{\varphi+1}{1-\alpha}) > 0$. Therefore:

$$d_\mu < 0, d_e > 0, d_\delta > 0.$$

Note that according to Campbell-Shiller decompsition, the return to core stock can be written as:

$$r_{s,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta d_{t+1},$$

where z_t is the log price-dividend ratio of the core stock. Postulate that $z_t = z_\mu \mu_t + z_e c_{e,t} + z_\delta \delta_t$, then the return to core stock is equal to:

$$r_{s,t+1} = \kappa_0 + \kappa_1(z_\mu \mu_{t+1} + z_e c_{e,t+1} + z_\delta \delta_{t+1}) - (z_\mu \mu_t + z_e c_{e,t} + z_\delta \delta_t) + \Delta d_{t+1}$$

We solve for the coefficients z_μ, z_e, z_δ from the Euler equation $E_t(m_{t+1} + r_{s,t+1}) + \frac{1}{2}var(m_{t+1} + r_{s,t+1}) = 0$.

$$E_t r_{s,t+1} = \kappa_0 + [(\kappa_1\rho_\mu - 1)z_\mu + (\rho_\mu - 1)d_\mu]\mu_t + [(\kappa_1\rho_e - 1)z_e + (\rho_e - 1)d_e]c_{e,t} + [(\kappa_1\rho_\delta - 1)z_\delta + (\rho_\delta - 1)d_\delta]\delta_t$$

And:

$$E_t m_{t+1} = m_\mu(1-\rho_\mu)\mu_t + m_e(1-\rho_e)c_{e,t} + m_\delta(1-\rho_\delta)\delta_t$$

Note that $var_t(m_{t+1} + r_{s,t+1})$ is a constant and the Euler equation has to hold for all values of state variables μ_t, we have the following:

$$(\kappa_1\rho_\mu - 1)z_\mu + (\rho_\mu - 1)d_\mu + (1-\rho_\mu)m_\mu = 0$$

We can solve for z_μ as:

$$z_\mu = \frac{1-\rho_\mu}{1-\kappa_1\rho_\mu}(m_\mu - d_\mu)$$

Similarly, the following equation should hold so that the Euler equation holds for all values of $c_{e,t}$:

$$(\kappa_1\rho_e - 1)z_e + (\rho_e - 1)d_e + (1-\rho_e)m_e = 0$$

Solve for z_e, z_δ as:

$$z_e = \frac{1-\rho_e}{1-\kappa_1\rho_e}(m_e - d_e), z_\delta = \frac{1-\rho_\delta}{1-\kappa_1\rho_\delta}(m_\delta - d_\delta)$$

Then we plug in the log price-dividend ratio into the Campbell-Shiller decomposition and solve for the core stock return:

$$r_{s,\mu} = \kappa_1 z_\mu + d_\mu = \frac{\kappa_1(1-\rho_\mu)}{1-\kappa_1\rho_\mu}(m_\mu - d_\mu) + d_\mu = \frac{\kappa_1(1-\rho_\mu)}{1-\kappa_1\rho_\mu}m_\mu + \frac{1-\kappa_1}{1-\kappa_1\rho_\mu}d_\mu < 0,$$

$$r_{s,e} = \frac{\kappa_1(1-\rho_e)}{1-\kappa_1\rho_e}m_e + \frac{1-\kappa_1}{1-\kappa_1\rho_e}d_e > 0, r_{s,\delta} = \frac{\kappa_1(1-\rho_\delta)}{1-\kappa_1\rho_\delta}m_\delta + \frac{1-\kappa_1}{1-\kappa_1\rho_\delta}d_\delta > 0.$$

B.4 Proof of Proposition 4

We use the nominal SDF of the households to price the nominal bonds. Consider the two-period bond issued at time t at price $P_t^{(2)}$. At time $t+1$, the bond price becomes $P_{t+1}^{(1)}$. Therefore, the following Euler equations hold for the two-period bond and one-period bond:

$$E_t M_{t+1}^{\$} P_{t+1}^{(1)} = P_t^{(2)}, E_t M_{t+1}^{\$} = P_t^{(1)}.$$

Take logs on both sides:

$$p_t^{(1)} = p_0^{(1)} + E_t m_{t+1}^{\$}$$

$$p_t^{(2)} = p_0^{(2)} + E_t m_{t+1}^{\$} + E_t p_{t+1}^{(1)}$$

where $p_0^{(1)}$ and $p_0^{(2)}$ are constants that originate from the second-order moments.

$$E_t m_{t+1}^{\$} = m_\mu(1-\rho_\mu)\mu_t + m_e(1-\rho_e)c_{e,t} + m_\delta(1-\rho_\delta)\delta_t$$

$$-\frac{1-\hat{\alpha}_c}{\phi}E_t(c_{c,t+1} - c_{c,t} - c_{e,t+1} + c_{e,t} - \delta_{t+1} + \delta_t) - E_t\pi_{t+1}$$

We rewrite $E_t m_{t+1}^{\$} = m_\mu^{\$}\mu_t + m_\delta^{\$}\delta_t + m_e^{\$}c_{e,t}$, where

$$m_\mu^{\$} = m_\mu(1-\rho_\mu) + \frac{1-\hat{\alpha}_c}{\phi}(1-\rho_\mu)c_{c,\mu} - \rho_\mu\pi_\mu,$$

$$m_e^{\$} = m_e(1-\rho_e) + \frac{1-\hat{\alpha}_c}{\phi}[(1-\rho_e)(c_{c,e}-1)] - \rho_e\pi_e, m_\delta^{\$} = m_\delta(1-\rho_\delta) + \frac{1-\hat{\alpha}_c}{\phi}[(1-\rho_\delta)(c_{c,\delta}-1)] - \rho_\delta\pi_\delta.$$

As $m_\mu < 0, c_{c,\mu} < 0, \pi_\mu > 0$, it is straightforward that $m_\mu^{\$} < 0$.

Recall that $m_e = \gamma(\hat{\alpha}_c c_{c,e} + 1 - \hat{\alpha}_c)$, thus

$m_e^{\$} = (1-\rho_e)\left[m_e + \frac{1-\hat{\alpha}_c}{\phi}(c_{c,e}-1)\right] = (1-\rho_e)(\eta_c c_{c,e} + \kappa_e) - \rho_e\pi_e < 0$

The last inequality comes from that fact that $\eta_c c_{c,e} + \kappa_e < 0$, which is from the proof of Proposition 1. Similarly, $m_\delta^{\$} < 0$.

The return to a long-term bond is equal to $r_{b,t+1} = p_{t+1}^{(1)} - p_t^{(2)} = const + E_{t+1}m_{t+2}^{\$} - (E_t m_{t+1}^{\$} + E_t p_{t+1}^{(1)})$. We decompose $r_{b,t+1}$ into predictable component $r_{b0,t}$ and unpredictable component $r_{b,\mu}\sigma_\mu \varepsilon_{t+1} + r_{b,e}\sigma_e \varepsilon_{e,t+1} + r_{b,\delta}\sigma_\delta \varepsilon_{\delta,t+1}$, we have:

$$r_{b,\mu} = m_\mu^{\$} < 0, r_{b,\delta} = m_\delta^{\$} < 0, r_{b,e} = m_e^{\$} < 0.$$

B.5 Proof of Proposition 5

The exposure of nominal foreign currency return to the three shocks are:

$$r_{fx,\mu} = \lambda_\mu + \frac{1-\hat{\alpha}_c}{\phi} c_{c,\mu} + \pi_\mu = \frac{\phi_\pi - 1}{\phi_\pi - \rho_\mu} \eta_c c_{c,\mu} < 0$$

$$r_{fx,e} = \lambda_e + \frac{1-\hat{\alpha}_c}{\phi}(c_{c,e} - 1) + \pi_e = \left(\eta_c + \frac{\lambda \kappa_c}{1-\beta\rho_e}\right) c_{c,e} + \left(\frac{\lambda}{1-\beta\rho_e} + 1\right)\kappa_e$$

$$r_{fx,\delta} = \lambda_\delta + \frac{1-\hat{\alpha}_c}{\phi}(c_{c,\delta} - 1) + \pi_\delta = \left(\eta_c + \frac{\lambda \kappa_c}{1-\beta\rho_\delta}\right) c_{c,\delta} + \left(\frac{\lambda}{1-\beta\rho_\delta} + 1\right)\kappa_\delta$$

Though we cannot determine conclusively on the signs of $r_{fx,e}$, we can see the three forces. Real SDF's loading on energy supply shock is $\lambda_e > 0$. The relative headline price loading is $\frac{1-\hat{\alpha}_c}{\phi}(c_{c,e} - 1) < 0$, and the core good inflation loading is $\pi_e > 0$. The signs of loadings on energy demand shock are similar.

B.6 Proof of Proposition 6

Commodity futures satisfy the following Euler equation:

$$E_t M_{t+1}^{\$} P_{e,t+1} \frac{P_{t+1}}{P_t} = E_t M_{t+1}^{\$} \frac{F_t}{P_{t-1}} \frac{P_{t-1}}{P_t}$$

Taking log on both sides:

$$f_t - p_{t-1} = f_0 + E_t(p_{e,t+1} + \pi_{t+1}) + \pi_t = f_0 + E_t\left[p_{e,t+1} + \pi_{t+1}\right] + \pi_t$$

$$= f_0 + \left(1 - \frac{1}{\phi}\right) \rho_\delta \delta_t - \frac{\rho_e}{\phi} c_{e,t} + \left[\frac{c_{c,\mu}}{\phi}\rho_\pi + (1+\rho_\pi)\pi_\mu\right] \mu_t$$

Straightforwardly, $f_e < 0, f_\delta > 0$. $\frac{c_{c,\mu}}{\phi}\rho_\pi + (1+\rho_\pi)\pi_\mu$ has an ambiguous sign, depending on whether the nominal effect of inflation or the change of relative energy price dominates.

C Additional Empirical Results

C.1 VAR Estimates

Table C1 reports the VAR coefficient matrix and the statistical significance.

Table C1: Shocks to Expected Inflation

	A. Risk Exposure		
	core	food	energy
core(-1)	0.46	0.16	1.74
	(7.41)	(1.11)	(2.15)
food(-1)	0.08	0.27	0.28
	(2.98)	(4.04)	(0.77)
inflation(-1)	0.01	0.02	-0.02
	(1.22)	(1.21)	(-0.29)
rf(-1)	0.15	0.01	0.00
	(3.01)	(0.05)	(-0.00)
pd(-1)	-1.23	-1.54	6.11
	(-3.19)	(-1.68)	(1.21)
output gap(-1)	0.06	0.32	0.31
	(1.31)	(2.90)	(0.50)
R^2	0.70	0.26	0.04

Notes: This table reports estimates of VAR(1). The t-statistics are in the parentheses. The VAR includes the core, food, and energy inflation, the risk-free rate, price-dividend ratio, and the output gap.

C.2 Inflation Expectation from Surveys

The survey of expectation has been widely used in studies of inflation. Though we focus on core and energy inflation separately, only core inflation is relatively persistent and predictable. Therefore, we use measures of headline inflation expectations as proxies for both headline and core inflation expectations. Specifically, we use the Survey of Professional Forecasters (SPF) and the Survey of Consumers, University of Michigan (MICH). The two survey series start at a later date, with the SPF starting in 1981Q3 and the MICH starting in 1978Q1.

The following tables show the results with the 8 average portfolios when we construct the headline and core inflation shocks as the difference between realized inflation and the inflation expectations. The results are largely similar with the ones presented in the main text, which confirms the robustness of our results.

Table C2: Inflation Expectations: Survey of Professional Forecasters

			A. Headline		B. Core and Energy			
	Mean	S.D.	headline β	t-stat	core β	t-stat	energy β	t-stat
Stock	8.83	16.40	0.87	(0.77)	-4.93	(-1.73)	0.26	(2.16)
Treasury	3.99	6.80	-2.57	(-6.15)	-3.00	(-2.72)	-0.19	(-4.17)
Agency	3.30	4.15	-1.39	(-5.30)	-2.09	(-3.03)	-0.08	(-2.90)
Corporate	4.49	5.17	-0.88	(-2.54)	-2.28	(-2.55)	-0.01	(-0.33)
Currency	1.76	7.05	1.04	(2.02)	0.39	(0.22)	0.12	(2.36)
Commodity	2.43	22.27	10.90	(8.74)	2.58	(0.83)	1.23	(9.35)
REIT	8.11	17.19	1.02	(0.87)	-7.22	(-2.46)	0.32	(2.60)
Intl Stock	7.59	16.21	0.60	(0.54)	-4.20	(-1.49)	0.24	(2.01)

C. Price of Risk

	8 Average Portfolios		38 Portfolios	
headline λ	0.22		0.09	
t-stat	(0.69)		(0.29)	
core λ		-1.28		-1.32
t-stat		(-2.24)		(-2.72)
energy λ		4.47		7.07
t-stat		(1.02)		(1.40)
R^2	0.43	0.84	0.40	0.73

Notes: Panels A and B of this table reports the regression results of the specification $r^e_{i,t} = \alpha_i + \beta^i_\pi \varepsilon_{\pi,t} + u_{i,t}$ for 8 average portfolios in each asset class. Panel A uses headline inflation shock as the risk factor. Panel B uses core and energy inflation jointly as risk factors. Panel C reports the price of risk estimates. The t-statistics are in the parentheses.

Table C3: Inflation Expectations: Surveys of Consumers, University of Michigan

			A. Headline		B. Core and Energy			
	Mean	S.D.	headline β	t-stat	core β	t-stat	energy β	t-stat
Stock	8.53	16.40	0.33	(0.32)	-2.81	(-1.74)	0.22	(1.91)
Treasury	2.93	7.50	-2.47	(-5.81)	-1.97	(-2.83)	-0.21	(-4.19)
Agency	2.47	5.11	-1.38	(-4.61)	-1.25	(-2.57)	-0.11	(-2.99)
Corporate	3.40	6.21	-1.17	(-3.13)	-1.70	(-2.79)	-0.04	(-0.89)
Currency	1.76	7.05	1.11	(2.24)	0.78	(0.72)	0.12	(2.35)
Commodity	2.34	21.91	8.89	(7.59)	0.81	(0.45)	1.16	(8.86)
REIT	7.96	17.46	-0.10	(-0.08)	-5.36	(-3.02)	0.29	(2.34)
Intl Stock	7.13	16.11	0.17	(0.17)	-2.57	(-1.61)	0.20	(1.75)

C. Price of Risk

	8 Average Portfolios		38 Portfolios	
headline λ	0.13		0.05	
t-stat	(0.36)		(0.15)	
core λ		-1.75		-1.57
t-stat		(-2.47)		(-2.76)
energy λ		3.60		8.26
t-stat		(0.93)		(1.93)
R^2	0.42	0.77	0.39	0.61

Notes: Panels A and B of this table reports the regression results of the specification $r^e_{i,t} = \alpha_i + \beta^i_\pi \varepsilon_{\pi,t} + u_{i,t}$ for 8 average portfolios in each asset class. Panel A uses headline inflation shock as the risk factor. Panel B uses core and energy inflation jointly as risk factors. Panel C reports the price of risk estimates. The t-statistics are in the parentheses.

C.3 GMM Estimation

In this section, we report the standard errors of prices of inflation risks with GMM estimation. The moment conditions are written as:

$$E_T \begin{bmatrix} (1 - b'\varepsilon_{\pi,t})r_t \\ \lambda - b'\varepsilon_{\pi,t}\varepsilon'_{\pi,t} \end{bmatrix} = 0$$

where $m_t = 1 - b'\varepsilon_{\pi,t}$ is the stochastic discount factor, $\varepsilon_{\pi,t}$ is the corresponding inflation risk factor, λ is the price of risk, and E_T is the operator of time-series average.

Table C4 reports the estimation results. These estimates are identical in sign and significance with the estimates obtained using the two-step Fama-MacBeth approach and very similar in magnitude.

Table C4: Price of Inflation Risks

	A. 8 Average Portfolios		B. 38 Portfolios	
headline λ	0.13		-0.12	
t-stat	(0.45)		(-0.48)	
core λ		-1.04		-1.07
t-stat		(-2.92)		(-3.77)
energy λ		3.86		3.81
t-stat		(1.37)		(1.77)

Notes: This table reports the price of risk estimated using the GMM. Panel A uses the 8 average portfolios from each asset class as test portfolios. Panel B uses the 38 portfolios as. In each panel, the first column reports the price of headline inflation and the second column reports the price of core and energy inflation. Standard errors are Newey-West adjusted.

C.4 The Use of PCE Inflation

Table C5 replaces the inflation measured as consumer price index with personal consumption expenditure (PCE) for the 8 average portfolios. All results are robust to the alternative measure of PCE inflation.

Table C5: Inflation Shocks: Personal Consumption Expenditure

			A. Headline		B. Core and Energy			
	Mean	S.D.	headline β	t-stat	core β	t-stat	energy β	t-stat
Stock	6.80	16.79	-1.42	(-1.08)	-5.31	(-2.32)	0.18	(1.58)
Treasury	2.07	6.90	-3.57	(-7.32)	-3.79	(-4.40)	-0.20	(-4.57)
Agency	2.44	5.10	-2.50	(-6.11)	-3.43	(-4.55)	-0.09	(-2.78)
Corporate	3.08	6.39	-2.28	(-4.57)	-3.89	(-4.35)	-0.04	(-0.99)
Currency	1.76	7.05	1.62	(2.28)	0.59	(0.44)	0.13	(2.47)
Commodity	4.47	21.90	10.46	(6.54)	0.42	(0.16)	1.07	(7.99)
REIT	7.96	17.46	1.06	(0.66)	-2.52	(-0.86)	0.22	(1.71)
Intl Stock	6.09	16.53	-0.95	(-0.72)	-4.55	(-1.93)	0.17	(1.46)

C. Price of Risk

	8 Average Portfolios		38 Portfolios	
headline λ	0.16		-0.04	
t-stat	(0.69)		(-0.23)	
core λ		-1.09		-0.95
t-stat		(-2.61)		(-3.71)
energy λ		5.83		6.12
t-stat		(1.60)		(1.82)
R^2	0.45	0.59	0.41	0.64

Notes: Panels A and B of this table reports the regression results of the specification $r^e_{i,t} = \alpha_i + \beta^i_\pi \varepsilon_{\pi,t} + u_{i,t}$ for 8 average portfolios in each asset class. Panel A uses headline inflation shock as the risk factor. Panel B uses core and energy inflation jointly as risk factors. Panel C reports the price of risk estimates. The t-statistics are in the parentheses.

C.5 Cash Flow News and Discount Rate News: Subsample Analysis

Table C6 presents the exposures to CF and DR news to core and energy infaltion before and after 1983. The year 1983 is usually treated as a change of regime from weak to strong monetary policy response to inflation (see, e.g., the estimated regimes in (Song, 2017)). Because of the more responsive monetary policy increases the discount rate, the DR news has a larger exposure in the latter sample.

Table C6: Cash Flow and Discount Rate News Exposures

	Cash Flow News				Discount Rate News			
	core	*t*-stat	energy	*t*-stat	core	*t*-stat	energy	*t*-stat
pre 1983	-2.03	(-3.65)	-0.05	(-0.41)	3.87	(2.89)	0.02	(0.07)
post 1983	-2.29	(-1.82)	0.00	(0.01)	4.91	(1.68)	-0.22	(-2.24)

Notes: This table reports the regression results of the cash flow and discount rate news on core and energy inflation shocks. The *t*-statistics are in the parentheses.

C.6 Shocks to Expected Inflation

This section reports the asset pricing test results for 7 average portfolios with respect to the shock to expected core inflation. The shock to expected core inflation is constructed as $A\varepsilon_{\pi,t}$, where A is the coefficient matrix in the VAR. The shock to expected core inflation is highly correlated with the shock to core inflation itself, so the asset pricing rests are similar, too. The correlation between shocks to core inflation and shocks to expected core inflation is 0.90.

Table C7: Shocks to Expected Inflation

	A. Risk Exposure			
	shock to core expectation	*t*-stat	energy shock	*t*-stat
Stock	-14.74	(-6.10)	0.36	(3.13)
Treasury	-5.03	(-5.21)	-0.16	(-3.50)
Agency	-4.98	(-5.78)	-0.05	(-1.49)
Corporate	-6.53	(-6.71)	0.01	(0.35)
Currency	-3.19	(-1.20)	0.16	(2.72)
Commodity	2.66	(0.86)	1.06	(7.61)
REIT	-14.06	(-4.16)	0.44	(3.44)
IntlStock	-14.29	(-5.74)	0.33	(2.94)

	C. Price of Risk	
	8 Average Portfolios	38 Portfolios
shock to core expectation	-0.40	-0.41
t-stat	(-2.88)	(-3.37)
energy shock	4.87	4.20
t-stat	(1.81)	(1.61)
R^2	0.95	0.84

Notes: This table reports the two-step asset pricing results for shock to core inflation expectation and energy shock. Panel A reports the asset return exposures for the 8 average portfolios, and Panel B reports the price of risk estimates for both 8 portfolios and 38 portfolios. The *t*-statistics are in the parentheses.

C.7 Betas at Lower Frequency

In this section, we report empirical results with low-frequency betas. This approach follows Bansal, Dittmar, and Lundblad (2005) and Lettau, Ludvigson, and Ma (2019). In the first-step of regression, we regress the cumulative asset return from quarter $t-\tau$ to quarter t on the unexpected inflation from quarter $t-\tau$ to quarter t.

$$r_{t-\tau,t} = a + \beta\varepsilon_{\pi,t-\tau,t} + u_t$$

The unexpected inflation over the τ quarters is computed from the VAR system $Y_t = c + Y_{t-1}A + u_t$.

$$\varepsilon_{\pi,t-\tau,t} = \sum_{j=0}^{\tau-1}(Y_{t-j} - E_{t-\tau}Y_{t-j}) = \sum_{j=0}^{\tau-1}Y_{t-j} - \tau c[(I-A)^{-1}(I-A^{\tau})] - \sum_{j=0}^{\tau-1}Y_{t-\tau}A^{\tau-j}$$

$$= \sum_{j=0}^{\tau-1}Y_{t-j} - \tau c[(I-A)^{-1}(I-A^{\tau})] - Y_{t-\tau}[(I-A)^{-1}(I-A^{\tau+1}) - I]$$

$\varepsilon_{\pi,t-\tau,t}$ are the corresponding rows of the vector. The covariation over longer-horizon captures the low-frequency relation between inflation and asset prices. $r_{t-\tau,t}$ is the cumulative asset excess return from quarter $t-\tau$ to t. We select $\tau = 8$ and examine the covariation at the eight-quarter frequency. Table C8 reports the β's prices of risks using the 8 average portfolios .

The asset loadings and price of risks are largely identical with the ones presented in the main text. The only difference is that the price of energy inflation is positive and statistically significant. One potential reason is that energy inflation is quite noisy at higher frequency, which contaminates the beta estimates and hinders the discovery of its price of risk. When we look at the lower-frequency covariation, energy inflation carries a positive risk premium, i.e., a higher energy inflation indicates good news for investors.

Table C8: Eight-Quarter Inflation Exposure and Price of Risk

A. Asset Return Exposure

	core	*t*-stat	energy	*t*-stat
Stock	-4.17	(-3.19)	0.40	(1.41)
Treasury	-1.45	(-2.16)	-0.20	(-2.77)
Agency	-2.10	(-6.94)	-0.11	(-2.57)
Corporate	-2.62	(-4.12)	-0.04	(-0.37)
Currency	-3.01	(-1.39)	0.24	(1.98)
Commodity	-4.64	(-3.09)	2.00	(6.07)
REIT	-1.76	(-0.59)	0.76	(1.77)
IntlStock	-4.68	(-3.78)	0.34	(0.92)

B. Price of Risk

	8 Average Portfolios	38 Portfolios
sticky λ	-1.19	-1.14
t-stat	(-2.86)	(-3.36)
flexible λ	1.50	3.16
t-stat	(0.73)	(1.35)
R^2	0.50	0.31

Notes: This table reports the two-step asset pricing results for core and energy inflation. Panel A reports the asset return exposures for the 8 average portfolios. The betas are computed by regressing 8-quarter cumulative excess returns on 8 quarter inflation shocks. Panel B reports the price of risk estimates for both 8 portfolios and 38 portfolios. The *t*-statistics are in the parentheses.

D The Extended Model

In this section, we present an extended version of the model in the main text. All asset pricing implications of the model carries through to the extended one. The goal of the extension is to break down the consumption CAPM. This way, the model is consistent with the empirical fact that controlling for consumption growth does not drive out the risk premium of core inflation.

D.1 Model Setting

The extended model is similar with the one in the main text in the preference on consumption, production technology, price stickiness and monopolistic competitive goods market structure. The only difference is that there are two types of agents, workers (fraction θ_w) and shareholders (fraction $1-\theta_w$). Workers supply labor and do not participate in the financial market. Shareholders own the equity claims of core firms. Energy goods are endowed. Variables with superscript w are associated with workers, and those with superscript e are associated with shareholders. We briefly outline the equilibrium conditions here.

D.1.1 Workers

The consumption-labor marginal optimality condition for workers is:

$$\frac{(C_t^w)^{-\gamma}}{P_t P_{ht}} = \frac{N_t^{\varphi}}{W_t}.$$

D.1.2 Shareholders

The Euler equations for shareholders are:

$$E_t \beta \left(\frac{C_{t+1}^s}{C_t^s} \right)^{-\gamma} \frac{P_{ht} P_t}{P_{h,t+1} P_{t+1}} (1 + i_t) = 1,$$

$$E_t \beta \left(\frac{C_{t+1}^s}{C_t^s} \right)^{-\gamma} \frac{P_{ht} P_t}{P_{h,t+1} P_{t+1}} \frac{D_{t+1} + P_{s,t+1}}{P_{s,t}} = 1.$$

D.1.3 Consumption Aggregation

For both workers and shareholders ($i = w, s$), the consumption basket is defined as:

$$C^i = \left[\alpha_c (C^i_c)^{\frac{\phi-1}{\phi}} + (1-\alpha_c)\left[\exp(\delta) C^i_e\right]^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}.$$

The relative price of energy satisfies:

$$P_e = \frac{1-\alpha_c}{\alpha_c}\left(\frac{C^i_e}{C^i_c}\right)^{-\frac{1}{\phi}} \exp\left(\frac{\phi-1}{\phi}\delta\right).$$

The headline price satisfies:

$$P_h = \left\{\alpha_c^{\phi} + (1-\alpha_c)^{\phi}\left[\exp(-\delta) P_e\right]^{1-\phi}\right\}^{\frac{1}{1-\phi}}.$$

D.1.4 Productive Core Firms

Each core firm produces one variety of core good and each firm is monopolistic in the specific variety production. All varieties are aggregated into core consumption in a CES manner, with elasticity of substitution ϕ. Production technology for variety j is:

$$Y_t(j) = A N_t(j)^{1-\alpha}.$$

The marginal cost of production, similar with the model in the main text, is:

$$MC(Y) = \frac{W}{P}\frac{1}{(1-\alpha)Y}\left(\frac{Y}{A}\right)^{\frac{1}{1-\alpha}}.$$

Since the aggregator labor supply in the economy is equal to $\theta_w N$, we normalize the aggregate TFP $A = \theta_w^{-(1-\alpha)}$. The New Keynesian Phillips Curve is written as

$$\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\frac{u'(C^s_{t+k})}{u'(C^s_t)}\frac{P_{ht}}{P_{h,t+1}}\left[Y_{t+k|t} + (P^*_t - \Psi'(Y_{t+k|t}))\frac{\partial Y_{t+k|t}}{\partial P^*_t}\right]\right\} = 0.$$

where:

$$\frac{\partial Y_{t+k|t}}{\partial P_t^*} = -\varepsilon_{t+k} \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon_{t+k}} \frac{C_{C,t+k}}{P_t^*} = -\varepsilon_{t+k} \frac{Y_{t+k|t}}{P_t^*}.$$

D.1.5 Market Clearing Conditions

There are two types of agents, so we need to include the aggregate budget constraint of one type of agent in the system of equations that consist of the equilibrium.

$$\theta_w (C_{ct}^w + P_{et} C_{et}^w) = \frac{W_t}{P_t} N_t \theta_w + P_{et} \theta_w Q_t.$$

The two market clearing conditions are:

$$\theta_w C_c^w + (1 - \theta_w) C_c^s = Y,$$

$$\theta_w C_e^w + (1 - \theta_w) C_e^s = C_e.$$

D.1.6 Monetary Policy

The monetary policy follows a Taylor rule:

$$i_t = \bar{i} + \phi_\pi \pi_t.$$

D.2 Log-linearization

When we log-linearize the system of equations, we make a parametric assumption to keep the algebra simplified: $\theta_w = (1 - \alpha) \exp(-\bar{\mu})$. At the steady state, labor income share is equal to $(1 - \alpha) \exp(-\bar{\mu})$. With our parametric assumption, the fraction of workers is equal to the steady state labor income share, the per capital consumption of core and energy goods are identical for workers and entrepreneurs at the steady state.

In the extended model, we make an additional parametric assumption that $\hat{\alpha}_c \to 1$. In the data energy inflation only accounts for about 10 percent of the headline inflation. This assumption can greatly simplify algebra in deriving the solutions.

The equilibrium of the economy satisfies the following log-linearized three-equation system with three unknowns: $c^s_{ct}, c^s_{et}, \pi_t$. The three equations are the Phillips curve, the Euler equation of the shareholders, and the workers' budget constraint.

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \mu_t + \lambda \left[\frac{\varphi + \alpha}{1-\alpha} y_t + \gamma \left(\hat{\alpha} c^w_{ct} + (1-\hat{\alpha}_c)(c^w_{et} + \delta_t) \right) + \frac{1-\hat{\alpha}_c}{\phi} (c^s_{ct} - c^s_{et} - \delta_t) \right]$$

$$-\gamma E_t \left[\hat{\alpha}_c (c^s_{c,t+1} - c^s_{ct}) + (1-\hat{\alpha}_c)(c^s_{e,t+1} - c^s_{et} + \delta_{t+1} - \delta_t) \right] - E_t \pi_{t+1}$$

$$-\frac{1-\hat{\alpha}_c}{\phi} E_t \Delta (c^s_{c,t+1} - c^s_{e,t+1} - \delta_{t+1}) + \phi_\pi \pi_t = 0,$$

$$\hat{\alpha}_c c^w_{ct} + (1-\hat{\alpha}_c) c^w_{et} = \hat{\alpha}_c \left[\gamma (\hat{\alpha}_c c^w_{ct} + (1-\hat{\alpha}_c)(c^w_{et} + \delta_t)) + \frac{\varphi + 1}{1-\alpha} y_t + \frac{1-\hat{\alpha}_c}{\phi} (c^s_{ct} - c^s_{et} - \delta_t) \right] + (1-\hat{\alpha}_c) c_{et},$$

where: $c^w_{ct} = \frac{1}{\theta_w}(c_{et} - c^s_{et}) + c^s_{ct}, c^w_{et} = \frac{1}{\theta_w}[c_{et} - (1-\theta_w)c^s_{et}], y_t = c_{et} - c^s_{et} + c^s_{ct}$, which can be straightforwardly derived from the relative price of energy good and the market clearing conditions.

D.3 Solution

We can express all the variables as linear functions of the three exogenous variables, μ_t, c_{et}, δ_t. Let:

$$c^s_{ct} = c_\mu \mu_t + c_e c_{et} + c_\delta \delta_t, c^s_{et} = e_\mu \mu_t + e_e c_{et} + e_\delta \delta_t, \pi_t = \pi_\mu \mu_t + \pi_e c_{et} + \pi_\delta \delta_t.$$

We keep the same assumption that the steady state level of markup is sufficiently large, and $\frac{1}{\gamma} > \phi > 1$.

D.3.1 Markup Shock

We can solve for the undetermined coefficients as follows:

$$c_\mu = \frac{\lambda}{(1-\beta\rho_\mu) y_\mu - \lambda \left[\frac{\varphi+\alpha}{1-\alpha}(1 - x_\mu) + \gamma(1 - \frac{x_\mu}{\theta_w}) \right]},$$

where:

$$x_\mu = \theta_w + \frac{\frac{\varphi+1}{1-\alpha}(\theta_w - \theta_w^2)}{-1+\gamma+\frac{\varphi+1}{1-\alpha}\theta_w} < 0, y_\mu = -\frac{\gamma(1-\rho_\mu)}{\phi_\pi - \rho_\mu} < 0,$$

$$e_\mu = x_\mu c_\mu, \pi_\mu = y_\mu c_\mu.$$

Therefore, $c_\mu < 0, e_\mu > 0, \pi_\mu > 0$.

The core output loading on the markup shock is equal to $y_\mu = c_\mu - e_\mu = c_\mu(1-x) < 0$.

D.3.2 Energy Demand Shock

$$\pi_\delta = \frac{\frac{\varphi+\alpha}{1-\alpha}\frac{1}{\gamma}(\frac{1}{\phi}-\gamma)}{(\frac{\varphi+\alpha}{1-\alpha}\frac{1}{\gamma}+1)\frac{\phi_\pi-\rho_\delta}{1-\rho_\delta}+\frac{1-\beta\rho_\delta}{\lambda}(1-\frac{\gamma}{1-\theta_w}-\frac{\theta_w}{1-\theta_w}\frac{\varphi+\alpha}{1-\alpha})}(1-\hat{\alpha}_c) > 0,$$

$$c_\delta = \frac{1}{\gamma}(\frac{1}{\phi}-\gamma)(1-\hat{\alpha}_c) - \frac{1}{\gamma}\frac{\phi_\pi-\rho_\delta}{1-\rho_\delta}\pi_\delta > 0,$$

$$e_\delta = -\frac{1-\beta\rho_\delta}{\lambda}\frac{\theta_w}{1-\theta_w}\pi_\delta < 0.$$

Thus, core output loading is equal to $y_\delta = c_\delta - e_\delta > 0$.

D.3.3 Energy Supply Shock

As in the main text, energy supply and demand shock plays exactly the same role, i.e.:

$$c_e > 0, \pi_e > 0, e_e - 1 < 0, y_e = c_e - (e_e - 1) > 0.$$

D.4 Dividend

In this section, we solve for the dividend loading on the three shocks. The dividend can be written as:

$$d_t = \frac{1}{1-\theta_w}[y_t - \theta_w(w_t - p_t - n_t)] - p_{ht}$$

$$= \frac{1}{1-\theta_w}\left[y_t - \theta_w\left(\frac{\varphi}{1-\alpha}y_t + \gamma(\hat{\alpha}_c c_{ct}^w + (1-\hat{\alpha}_c)(c_{et}^w + \delta_t) + \frac{1-\hat{\alpha}_c}{\phi}(c_{ct}^s - c_{et}^s - \delta_t) + \frac{1}{1-\alpha}y_t\right)\right]$$

$$-\frac{1-\hat{\alpha}_c}{\phi}(c_{ct}^e - c_{et}^e - \delta_t).$$

Apply the assumption that $\hat{\alpha}_c$ being close to 1, we can derive

$$d_\mu = \frac{1}{1-\theta_w}\left[\left(1 - \frac{(\varphi+1)\theta_w}{1-\alpha}\right)(c_\mu - e_\mu) - \theta_w\gamma(c_\mu - \frac{1}{\theta_w}e_\mu)\right] < 0,$$

$$d_\delta = \left[1 - \theta_w\left(\frac{\varphi+1}{1-\alpha} + \gamma\right)\right]c_\delta + \left(-1 + \gamma + \frac{\varphi+1}{1-\alpha}\theta_w\right)e_\delta + \theta_w\left(\frac{1}{\phi} - \gamma\right)(1-\hat{\alpha}_c) + \frac{(1-\theta_w)(1-\hat{\alpha}_c)}{\phi}$$

$$> (1-\theta_w)\gamma c_\delta + \theta_w\left(\frac{1}{\phi} - \gamma\right)(1-\hat{\alpha}_c)\frac{(1-\theta_w)(1-\hat{\alpha}_c)}{\phi} > 0.$$

The inequality comes from the fact that $\theta_w < \frac{(1-\alpha)(1-\gamma)}{1+\varphi}$. The sign of d_e is the same as d_δ.

D.5 The Stochastic Discount Factor

The stochastic discount factor is written as:

$$m_{t+1} = m_\mu(1-\rho_\mu)\mu_t + m_e(1-\rho_e)c_{et} + m_\delta(1-\rho_\delta)\delta_t - \lambda_\mu\sigma_\mu\varepsilon_{\mu,t+1} - \lambda_e\sigma_e\varepsilon_{e,t+1} - \lambda_\delta\sigma_\delta\varepsilon_{\delta,t+1}.$$

In this model, only the shareholders' consumption matters for asset pricing:

$$m_{t+1} = -\gamma(c_{t+1}^s - c_t^s) = -\gamma[\hat{\alpha}_c(c_{c,t+1}^s - c_{c,t}^s) + (1-\hat{\alpha}_c)(c_{e,t+1}^s - c_{e,t}^s + \delta_{t+1} - \delta_t)].$$

It is straightforward to derive that:

$$\lambda_\mu = \gamma c_\mu < 0, \lambda_\delta = \gamma c_\delta + \gamma(1-\hat{\alpha}_c)(e_\delta + 1) > 0, \lambda_e = \gamma c_e + \gamma(1-\hat{\alpha}_c)e_e > 0,$$

$$m_\mu = \gamma c_\mu < 0, m_\delta = \gamma c_\delta + \gamma(1-\hat{\alpha}_c)(e_\delta + 1) > 0, m_e = \gamma c_e + \gamma(1-\hat{\alpha}_c)e_e > 0.$$

D.6 Nominal SDF and Asset Prices

From the main text, we see that returns to core stocks, currencies, and commodities only depend on loadings of dividend and SDF on the three shocks. All derivations in the main text apply in the extended model. For bond returns, we need to derive the asset return loadings using the nominal SDF. We derive the loadings of nominal SDFs here as well, $E_t m_{t+1}^{\$} = E_t m_{t+1} - E_t \pi_{t+1} - E_t(p_{h,t+1} - p_{h,t})$.

$$m_\mu^{\$} = m_\mu(1-\rho_\mu) - \rho_\mu \pi_\mu + \frac{1-\hat{\alpha}_c}{\phi}(c_\mu - e_\mu)(1-\rho_\mu) < 0,$$

$$m_\delta^{\$} = m_\delta(1-\rho_\delta) - \rho_\delta \pi_\delta + \frac{1-\hat{\alpha}_c}{\phi}(c_\delta - e_\delta - 1)(1-\rho_\delta)$$

$$= (1-\rho_\delta)\left[\gamma c_\delta + (1-\hat{\alpha}_c)\left(\gamma - \frac{1}{\phi}\right) - \rho_\delta \pi_\delta\right].$$

Since $-\gamma c_\delta + \left(\frac{1}{\phi} - \gamma\right)(1-\hat{\alpha}_c) = \frac{\phi_\pi - \rho_\delta}{1-\rho_\delta}\pi_\delta$, $m_\delta^{\$} = (1-\rho_\delta)\left(-\rho_\delta - \frac{\phi_\pi - \rho_\delta}{1-\rho_\delta}\right)\pi_\delta < 0$. Similarly, $m_e^{\$} < 0$.

Since the sign of real and nominal SDF loadings and dividend loadings are exactly the same as in the model presented in the main text, all asset return loadings are identical, too. We skip all the derivations of asset prices here for the extended model.